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PII: S0393-0440(17)30169-9
DOI: http://dx.doi.org/10.1016/j.geomphys.2017.06.013
Reference: GEOPHY 3024

To appear in: *Journal of Geometry and Physics*

Received date : 30 October 2014
Revised date : 19 April 2017
Accepted date : 27 June 2017

Please cite this article as: A. Baklouti, Quadratic Hom-Lie triple systems, *Journal of Geometry and Physics* (2017), http://dx.doi.org/10.1016/j.geomphys.2017.06.013

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Abstract

Hom-Lie triple systems endowed with a symmetric invariant nondegenerate bilinear form are called quadratic Hom-Lie triple systems. In this work, we introduce the notion of double extension of Hom-Lie triple systems so that we can give an inductive description of quadratic Hom-Lie triple systems.

Key words: Representation, quadratic Lie triple systems, Hom-Lie triple systems, central extension, double extension.

MSC: 17A40, 17B05.

Introduction

Using the Bianchi identities, K. Nomizu [23] characterized, by some identities involving the torsion and the curvature, reductive homogeneous spaces with some canonical connection. K. Yamaguti [26] gave an algebraic interpretation of these identities by considering the torsion and curvature tensors of Nomizus canonical connection as a bilinear and a trilinear algebraic operations satisfying some axioms, and thus defined what he called a general Lie triple system. A Lie triple system as it is considered in his paper is a finite-dimensional linear space $\mathbf{L}$ together with a trilinear map $[-,-,-]: \mathbf{L}\times \mathbf{L}\times \mathbf{L}\to \mathbf{L}$ which satisfies the following identities:

$$[x,x,z]=0$$
$$[x,y,z]+[y,z,x]+[z,x,y]=0$$
$$[u,v,[x,y,z]]=[[u,v,x],y,z]+[x,[u,v,y],z]+[x,y,[u,v,z]], \forall x,y,z,u,v\in \mathbf{L}.$$

If $\mathbf{L}$ is equipped with a symmetric nondegenerate bilinear form B which satisfies $B([x,y,z],u)=B(z,[y,x,u])$ for all $x,y,z\in \mathbf{L}$, we say that $\mathbf{L}$ is quadratic.

In physics, quadratic Lie triple systems appear in many different contexts. Let us mention a few of them. Lie triple systems can be used to construct solutions of the Yang-Baxter equation which appears in many subjects from statistical mechanics, exactly solvable two-dimensional field theory, the quantum