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2-CAPABILITY AND 2-NILPOTENT MULTIPLIER OF FINITE DIMENSIONAL NILPOTENT LIE ALGEBRAS

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Abstract. In the present context, we investigate to obtain some more results about 2-nilpotent multiplier $\mathcal{M}^{(2)}(L)$ of a finite dimensional nilpotent Lie algebra L. For instance, we characterize the structure of $\mathcal{M}^{(2)}(H)$ when H is a Heisenberg Lie algebra. Moreover, we give some inequalities on dim $\mathcal{M}^{(2)}(L)$ to reduce a well known upper bound on 2-nilpotent multiplier as much as possible. Finally, we show that $H(m)$ is 2-capable if and only if m=1.

1. Introduction

For a finite group G, let G be the quotient of a free group F by a normal subgroup R, then the c-nilpotent multiplier $\mathcal{M}^{(c)}(G)$ is defined as

$$R \cap \gamma_{c+1}(F)/\gamma_{c+1}[R, F],$$

in which $\gamma_{c+1}[R, F] = [\gamma_c[R, F], F]$ for $c \geq 1$. It is an especial case of the Baer invariant [3] with respect to the variety of nilpotent groups of class at most c. When $c = 1$, the abelian group $\mathcal{M}(G) = \mathcal{M}^{(1)}(G)$ is more known as the Schur multiplier of G and it is much more studied, for instance in [11, 14, 18].

Since determining the c-nilpotent multiplier of groups can be used for the classification of group into isoclinism classes(see [2]), there are multiple papers concerning this subject.

Recently, several authors investigated to develop some results on the group theory case to Lie algebra. In [22], analogues to the c-nilpotent multiplier of groups, for a given Lie algebra L, the c-nilpotent multiplier of L is defined as

$$\mathcal{M}^{(c)}(L) = R \cap F^{c+1}/[R, F]^{c+1},$$

in which L presented as the quotient of a free Lie algebra F by an ideal R, $F^{c+1} = \gamma_{c+1}(F)$ and $[R, F]^{c+1} = \gamma_{c+1}[R, F]$. Similarly, for the case $c = 1$, the abelian Lie algebra $\mathcal{M}(L) = \mathcal{M}^{(1)}(L)$ is more studied by the first author and the others (see for instance [4, 5, 6, 8, 9, 10, 15, 16, 17, 24]).

The c-nilpotent multiplier of a finite dimensional nilpotent Lie algebra L is a new field of interest in literature. The present context is involving the 2-nilpotent multiplier of a finite dimensional nilpotent Lie algebra L. The aim of the current paper is divided into several steps. In [22, Corollary 2.8], by a parallel result to the group theory result, showed for every finite nilpotent Lie algebra L, we have

$$\dim(\mathcal{M}^{(2)}(L)) + \dim(L^3) \leq \frac{1}{3}n(n-1)(n+1). \tag{1.1}$$

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