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Abstract

Certain dissipative physical systems closely resemble Hamiltonian systems in $\mathbb{R}^{2n}$, but with the canonical equation for one of the variables in each conjugate pair rescaled by a real parameter. To generalise these dynamical systems to symplectic manifolds in this paper we introduce and study the properties of deformed Hamiltonian vector fields on Lagrangian fibrations. We describe why these objects have some interesting applications to symplectic geometry and discuss how their physical interpretation motivates new problems in mathematics.

Keywords: Symplectic geometry, Dissipative systems

1. Introduction

Symplectic geometry arises as the natural generalisation of conservative Hamiltonian mechanics to differentiable manifolds. The phase space of a Hamiltonian system is generalised to a symplectic manifold and phase portraits are interpreted as integral curves of a Hamiltonian vector field. Symplectic geometry therefore has its origins in classical physics, but more recent times have seen string theory play a role in the discovery of Gromov-Witten invariants and the birth of Floer theory. Together with mirror symmetry, these developments are some of the great success stories of symplectic geometry that can be partially attributed to mathematical physics. Certain dissipative dynamical systems arising in physics are also described using a symplectic viewpoint although not in the setting of differentiable manifolds [1, 2]. The current paper grew out of an attempt to put these dynamical systems into the context of symplectic geometry.

Hamiltonian vector fields, which generalise dynamical systems appearing in classical mechanics, play a central role in several different versions of Floer theory for symplectic manifolds and

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