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EICHLER-SHIMURA ISOMORPHISM FOR COMPLEX HYPERBOLIC LATTICES

INKANG KIM AND GENKAI ZHANG

ABSTRACT. We consider the cohomology group $H^1(\Gamma, \rho)$ of a discrete subgroup $\Gamma \subset G = SU(n, 1)$ and the symmetric tensor representation ρ on $S^m(\mathbb{C}^{n+1})$. We give an elementary proof of the Eichler-Shimura isomorphism that harmonic forms $H^1(\Gamma\backslash G/K, \rho)$ are $(0, 1)$-forms for the automorphic holomorphic bundle induced by the representation $S^m(\mathbb{C}^n)$ of K.

1. INTRODUCTION

Let B be the unit ball in $\mathbb{C}^n$ considered as the Hermitian symmetric space $B = G/K$ of $G = SU(n, 1)$, $n > 1$. Let Γ be a cocompact torsion free discrete subgroup of G and ρ a finite dimensional representation of G, and $X = \Gamma\backslash B$. The representation ρ of G defines also one for $\Gamma \subset G$. The first cohomology $H^1(\Gamma, \rho)$ is of substancial interests and appears naturally in the study of infinitesimal deformation of Γ in a bigger group $G' \supset G$; see [8, 5, 2]. It is a classical result of Raghunathan [14] that the cohomology group $H^1(\Gamma, \rho)$ vanishes except when $\rho = \rho_m$ is the symmetric tensor $S^m(\mathbb{C}^{n+1})$ (or ρ'_m on $S^m(\mathbb{C}^{n+1})'$). In a recent work [8] it is proved that realizing $H^1(X, \rho)$ as harmonic forms, it consists of $(0, 1)$-forms for the symmetric tensor of the holomorphic tangent bundle of $X = \Gamma\backslash B$. The proof in [8] uses a Hodge vanishing theorem and the Koszul complex. In the present paper we shall give a rather elementary proof of the result. We will prove that any harmonic form with values in $S^m(\mathbb{C}^{n+1})$ is $(0, 1)$-form taking values in $S^m(\mathbb{C}^n)$. Let TX and $T'X$ be the holomorphic tangent and cotangent bundles respectively. Let $\mathcal{L}^{-1}$ be the line bundle on X defined so that $\mathcal{L}^{-(n+1)}$ is the canonical line bundle $\mathcal{K} = K_X$. More precisely we shall prove the following, the notations being explained in §2,

Theorem 1.1. *Let Γ be a torsion free cocompact lattice of G acting properly discontinuously on B.*

(1) *Let $\alpha \in A^1(\Gamma, B, \rho_m)$ be a harmonic form. Then α is a $(0, 1)$-form on $\Gamma\backslash B$ with values in the holomorphic vector bundle $S^mTX \otimes \mathcal{L}^{-m}$.*

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