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Quantum state geometry and entanglement of two spins with anisotropic interaction in evolution

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Quantum evolution of a two-spin system with anisotropic Heisenberg Hamiltonian in the magnetic field is considered. We show that this evolution happens on some manifold with geometry depending on the ratio between the interaction couplings and on the initial state. The Fubini-Study metric of this manifold is calculated. The entanglement of the states belonging to this manifold is examined. Also we investigate similar problem for a two-spin system described by the Dzyaloshinsky-Moria Hamiltonian. The problem is solved by using the fact that this Hamiltonian and the anisotropic Heisenberg Hamiltonian are linked by the unitary transformation.

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1 Introduction

Understanding of the geometry of quantum state manifolds is useful in the study of many problems related to the quantum evolution. For example, using the fact that the whole space of states of a two-level system is represented by the Bloch sphere, we easily obtain the trajectory of quantum evolution between two states of this system (see, for instance, [1, 2, 3, 4, 5, 6, 7]). This trajectory is a curve between two points on the sphere. The minimal trajectory of a spin-1/2 particle, driven by a magnetic field, was found, using geometrical properties of the quantum state space [6, 7]). Also, in a similar way, the quantum brachistochrone problem for an arbitrary spin in a magnetic field [8] and the Zermelo navigation problem [9, 10, 11, 12, 13] were solved. Another well-known problem, where the understanding of the geometry of the quantum states space plays an important role, is the problem of finding the time-optimal Hamiltonian which provides the evolution between two specified quantum states (the quantum brachistochrone problem) [14]. This problem was solved by using symmetry properties of the quantum state space. Also it was shown that the problem of finding of the quantum circuit of unitary operations which provide time-optimal evolution on a system of qubits [15, 16, 17, 18] and qutrits [19] is related to the problem of finding of the minimal distance between two points on a Riemannian metric. One can