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Non-existence of natural states for Abelian Chern-Simons theory

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Abstract

We give an elementary proof that Abelian Chern-Simons theory, described as a functor from oriented surfaces to C^*-algebras, does not admit a natural state. Non-existence of natural states is thus not only a phenomenon of quantum field theories on Lorentzian manifolds, but also of topological quantum field theories formulated in the algebraic approach.

1 Introduction and summary

A locally covariant quantum field theory (LCQFT) [BFV03] is a functor $\mathcal{A} : \mathsf{Loc} \to C^*\mathsf{Alg}$ from a category of (globally hyperbolic Lorentzian) spacetimes to the category of C^*-algebras satisfying suitable physical axioms. The C^*-algebra $\mathcal{A}(M)$ assigned to a spacetime M is interpreted as the algebra of quantum observables which can be measured in M. The C^*-algebra homomorphism $\mathcal{A}(f) : \mathcal{A}(M) \to \mathcal{A}(M')$ assigned to a spacetime embedding $f : M \to M'$ allows us to associate observables in larger spacetimes starting from observables in smaller ones.

For a quantum physical interpretation of a LCQFT $\mathcal{A} : \mathsf{Loc} \to C^*\mathsf{Alg}$ it is necessary to choose for each spacetime M a state $\omega_M : \mathcal{A}(M) \to \mathbb{C}$ on the C^*-algebra $\mathcal{A}(M)$, i.e. a positive, normalized and continuous linear functional. (The GNS-representation then leads to the usual Hilbert space formulation of quantum physics.) Motivated by the functorial structure of LCQFT, it seems natural to demand that the family of states $\{\omega_M\}_{M\in\mathsf{Loc}}$ is compatible with the functor $\mathcal{A} : \mathsf{Loc} \to C^*\mathsf{Alg}$ in the sense that

$$\omega_{M'} \circ \mathcal{A}(f) = \omega_M \ , \tag{1.1}$$

for all Loc-morphisms $f : M \to M'$. Such compatible families of states are called *natural states* on $\mathcal{A} : \mathsf{Loc} \to C^*\mathsf{Alg}$.

Even though the idea of natural states is very beautiful and appealing, there are hard obstructions to the existence of natural states. Early arguments were already given by Brunetti, Fredenhagen and Verch in [BFV03]. Later, a no-go theorem on the existence of natural states (under some additional assumptions) has been proven by Fewster and Verch in [FV12]. This no-go theorem makes use of very particular properties of dynamical quantum field theories on Lorentzian spacetimes, e.g. the concept of relative Cauchy evolution.