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SPINORIAL REPRESENTATION OF SUBMANIFOLDS IN METRIC LIE GROUPS

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Abstract. In this paper we give a spinorial representation of submanifolds of any dimension and codimension into Lie groups equipped with left invariant metrics. As applications, we get a spinorial proof of the Fundamental Theorem for submanifolds into Lie groups, we recover previously known representations of submanifolds in $\mathbb{R}^n$ and in the 3-dimensional Lie groups $\mathbb{S}^3$ and $E(\kappa,\tau)$, and we get a new spinorial representation for surfaces in the 3-dimensional semi-direct products: this achieves the spinorial representations of surfaces in the 3-dimensional homogeneous spaces. We finally indicate how to recover a Weierstrass-type representation for CMC-surfaces in 3-dimensional metric Lie groups recently given by Meeks, Mira, Perez and Ros.

Keywords: Spin geometry, metric Lie groups, isometric immersions, Weierstrass representation.

2010 Mathematics Subject Classification: 53C27, 53C40.

1. Introduction

The purpose of this paper is to give a spinorial representation of an isometric immersion of a Riemannian manifold M into a Lie group G equipped with a left invariant metric. The result is roughly the following: if M is a simply connected Riemannian manifold, E is a real vector bundle on M equipped with a fiber metric and a compatible connection, and $B : TM \times TM \to E$ is bilinear and symmetric, then an isometric immersion of M into G with normal bundle E and second fundamental form B is equivalent to the existence of a spinor field φ solution of a Killing-type equation on M; the spinor bundle of G is constructed from the Clifford algebra of the metric Lie algebra $\mathcal{G}$ of the group, and the immersion is explicitly obtained by the integration of a $\mathcal{G}$-valued 1-form on M defined in terms of the spinor field φ. We state here the main result and refer to Section 2 for the precise definitions of the spinor bundle and the various objects defined on it.

Theorem 1. *Let M be a simply connected Riemannian manifold, E a real vector bundle on M equipped with a fiber metric and a compatible connection, and $B : TM \times TM \to E$ a bilinear and symmetric map. We assume that E and TM are oriented and spin, with given spin structures and that the compatibility conditions (21) and (22) are statisfied. Then, the following statements are equivalent:*

(1) There exists a section $\varphi \in \Gamma(U\Sigma)$ such that

$$\nabla_X \varphi = -\frac{1}{2}\sum_{j=1}^{p} e_j \cdot B(X, e_j) \cdot \varphi + \frac{1}{2}\Gamma(X) \cdot \varphi \tag{1}$$

for all $X \in TM$.