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A.A. Matveeva, V.A. Poberezhny

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Two-dimensional Riemann problem for rigid representations on an elliptic curve

A. A. Matveeva[1] and V. A. Poberezhny[1,2]

[1] *National Research University Higher School of Economics, Russian Federation*
[2] *Institute for Theoretical and Experimental Physics*

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Abstract

We consider a generalization of Riemann-Hilbert problem on elliptic curves. For a given elliptic curve and irreducible representation of free group with two generators we construct explicitly a semistable vector bundle of degree zero obeying a logarithmic connection such that its monodromy over fundamental parallelogram is equivalent to given free group representation, monodromy along $a-$cycle is trivial and monodromy along $b-$cycle belong to certain orbit.

Introduction

The classical Riemann-Hilbert problem deals with Fuchsian systems on the Riemann sphere. It explores the existence of Fuchsian system of linear differential equations with given singular points and prescribed monodromy representation. In general setting the problem has negative solution [1]. There also exist certain sufficient conditions for positive solvability. The problem can be solved positively for irreducible monodromy representations, representations of dimension two and some other cases.

There are different approaches to generalization of the problem on other than Riemann sphere compact Riemann surfaces, see [2, 3, 4]. In this paper we follow the formulation proposed in [3] appealing to geometric approach to the problem. A Fuchsian system on sphere can be considered as a logarithmic connection in trivial vector bundle on sphere. It appears that in this approach essential properties of trivial vector bundle are the semi-stability and equality of its degree to zero [5]. The trivial bundle appears here because on Riemann sphere in any dimension it is the only holomorphic semi-stable vector bundle of degree zero. To this reason the generalization that we consider is being given elliptic curve, set of points on it and representation of fundamental group to construct over that curve a semi-stable vector bundle of degree zero equipped with logarithmic connection having prescribed singular points and required monodromy representation.

On the Riemann sphere positive results on classical Riemann-Hilbert problem are usually the existence theorems. There are rather few cases when the solution can be constructed explicitly. Among them are $(p = 2, n = 3)$ case of rank two systems with three singular points studied by Krylov [6] who proved complete positive solvability in that case, quasi-permutation monodromy in general position, studied by Korotkin [7], $(p = 2, n = 4)$ case for some special subgroups of $\mathrm{SL}(2, \mathbb{C})$ arising from algebraic solutions of $\mathrm{P_{VI}}$ equation studied by Boalch [8] and some other special cases [9].