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POISSON-RIEMANNIAN GEOMETRY

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Abstract. We study noncommutative bundles and Riemannian geometry at the semiclassical level of first order in a deformation parameter λ, using a functorial approach. This leads us to field equations of 'Poisson-Riemannian geometry' between the classical metric, the Poisson bracket and a certain Poisson-compatible connection needed as initial data for the quantisation of the differential structure. We use such data to define a functor Q to $O(\lambda^2)$ from the monoidal category of all classical vector bundles equipped with connections to the monoidal category of bimodules equipped with bimodule connections over the quantized algebra. This is used to 'semiquantize' the wedge product of the exterior algebra and in the Riemannian case, the metric and the Levi-Civita connection in the sense of constructing a noncommutative geometry to $O(\lambda^2)$. We solve our field equations for the Schwarzschild black-hole metric under the assumption of spherical symmetry and classical dimension, finding a unique solution and the necessity of nonassociativity at order λ^2, which is similar to previous results for quantum groups. The paper also includes a nonassociative hyperboloid, nonassociative fuzzy sphere and our previously algebraic bicrossproduct model.

1. Introduction

Noncommutative geometry has been successful in recent years in extending notions of geometry to situations where the 'coordinate algebra' is noncommutative. Such algebras could arise on quantisation of the phase space in the passage from a classical mechanical system to a quantum one, in which case noncommutative geometry allows us to understand the deeper geometry of such systems. An example here is the quantum Hall effect[13, 31]. It is also now widely accepted that noncommutative Riemannian geometry of some kind should be a more accurate description of spacetime coordinates so as to include the effects of quantum corrections arising out of quantum gravity. The deformation parameter in this case is not expected to be Planck's constant but the Planck scale λ_P. The main evidence for such a *quantum spacetime hypothesis* is by analogy with 3D quantum gravity, see e.g. [36], but the hypothesis has also been extensively explored in specific models such as the bicrossproduct one[35], with key implications such as variable speed of light[2] and frequency dependent gravitational time dilation[33].

These noncommutative models have, however, all been constructed on a case by case basis using algebraic methods and there has so far been no fully systematic 'quantisation method' that takes wider geometrical semiclassical data and quantizes it in the same manner as we are used to for the algebra alone, although there have been some early steps in this direction notably concerning quantizing vector bundles[29, 15, 23, 27] as well as later works including our own[25, 26, 5, 6, 7] and recent works such as [9, 10, 4]. We recall that for the noncommutative algebra alone the semiclassical data is well-known to be a Poisson bracket and in this case the quantisation problem was famously solved to all orders in deformation theory by Kontsevich[30] and more explicitly in the symplectic case by Fedosov[22]. The question we address is *what exactly is the semiclassical*

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