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PII: S0393-0440(16)30157-7
DOI: http://dx.doi.org/10.1016/j.geomphys.2016.06.016
Reference: GEOPHY 2791

To appear in: *Journal of Geometry and Physics*

Received date: 18 March 2016
Revised date: 28 May 2016
Accepted date: 29 June 2016

Please cite this article as: S. Jiménez, J.J. Morales-Ruiz, R. Sánchez-Cauce, M.-A. Zurro, Differential Galois theory and Darboux transformations for Integrable Systems, *Journal of Geometry and Physics* (2016), http://dx.doi.org/10.1016/j.geomphys.2016.06.016

Differential Galois Theory and Darboux transformations for Integrable Systems

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Abstract

We apply the Differential Galois Theory of linear partial differential systems to the Bäcklund–Darboux transformations of the AKNS solitonic partial differential equations. We prove that the Galois group of the transformed system is isomorphic to a subgroup of the Galois group of the initial system. As an example, we study the integrability in closed form of the linear systems corresponding to the solitonic solutions of KdV equation.

Keywords: Differential Galois Theory, Picard-Vessiot Theory, Integrability, Darboux Transformations, Solitons
2010 MSC: Primary: 12H05, Secondary: 35Q51, 37K10

Introduction

In the last years the differential Galois theories, i.e., the Galois theories of differential equations, has undergone an important renaissance, partially due to their relevance in the applications to other areas, like integrability of dynamical systems, connections with asymptotic theory (Stokes multipliers), some special spectral problems and to the integrability–reducibility of the Painlevé transcendents.

Picard–Vessiot theory is the Galois theory for linear differential equations. This will be essentially the only differential Galois theory relevant in our paper. It was initiated by Picard and Vessiot at the end of the nineteen century for ordinary linear differential equations. Then it was formalized by Kolchin in the last century. In particular, Picard–Vessiot theory for fields with several derivations, necessary for systems of linear differential equations, was constructed by Kolchin in the fifties of the last century ([17]). Some years later, Kolchin himself introduced a more general differential Galois theory, given by the so-called strongly normal extensions, where this Picard-Vessiot theory is included in a natural way. This culminated in the publication in the seventies of the monograph [18]. In his constructions Kolchin uses the algebraic geometry of Weil, but in the first decade of the present century, Kovacic reformulates Kolchin's strongly normal extensions in the language of differential schemes, instead of Weil's geometry (see [19] and references therein). Three

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