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EXAMPLES OF INTEGRABLE AND NON-INTEGRABLE SYSTEMS ON SINGULAR SYMPLECTIC MANIFOLDS

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Abstract. We present a collection of examples borrowed from celestial mechanics and projective dynamics. In these examples symplectic structures with singularities arise naturally from regularization transformations, Appell's transformation or classical changes like McGehee coordinates, which end up blowing up the symplectic structure or lowering its rank at certain points. The resulting geometrical structures that model these examples are no longer symplectic but symplectic with singularities which are mainly of two types: b^m-symplectic and m-folded symplectic structures. These examples comprise the three body problem as non-integrable exponent and some integrable reincarnations such as the two fixed-center problem. Given that the geometrical and dynamical properties of b^m-symplectic manifolds and folded symplectic manifolds are well-understood [GMP, GMP2, GMPS, KMS, KM, Ma, CGP, GL, GLPR, MO, S, GMW], we envisage that this new point of view in this collection of examples can shed some light on classical long-standing problems concerning the study of dynamical properties of these systems seen from the Poisson viewpoint.

1. Introduction

Integrability and non-integrability of some classical problems in physics and celestial mechanics such as the Kepler problem of the 2 or 3-body problems is well-understood [AKN]. Even if the 3-body problem is not integrable some restricted cases like the 2-fixed center problem are integrable. When studying such systems ad hoc transformations have been considered in order to understand their dynamics (for instance in the McGehee change of coordinates) and integrability (Appell's transformation for Newton's systems).

In this article we provide a list of classical examples in celestial mechanics and projective dynamics and analyze the classical changes done in the theory to study their geometrical and dynamical properties. We observe that these classical changes induce singularities in the Darboux symplectic structure of the phase space (cotangent bundle). Transformations that preserve

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