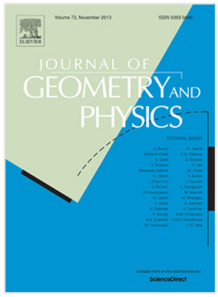

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Finsler geometry of nonlinear elastic solids with internal structure

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Abstract

Concepts from Finsler differential geometry are applied towards a theory of deformable continua with internal structure. The general theory accounts for finite deformation, nonlinear elasticity, and various kinds of structural features in a solid body. The general kinematic structure of the theory includes macroscopic and microscopic displacement fields–i.e., a multiscale representation– whereby the latter are represented mathematically by the director vector of pseudo-Finsler space, not necessarily of unit magnitude. A physically appropriate fundamental (metric) tensor is introduced, leading to affine and nonlinear connections. A deformation gradient tensor is defined via differentiation of the macroscopic motion field, and another metric indicative of strain in the body is a function of this gradient. A total energy functional of strain, referential microscopic coordinates, and horizontal covariant derivatives of the latter is introduced. Variational methods are applied to derive Euler-Lagrange equations and Neumann boundary conditions. The theory is shown to encompass existing continuum physics models such as micromorphic, micropolar, strain gradient, phase field, and conventional nonlinear elasticity models, and it can reduce to such models when certain assumptions on geometry, kinematics, and energy functionals are imposed. The theory is applied to analyze two physical problems in crystalline solids: shear localization/fracture in a two-dimensional body and cavitation in a spherical body. In these examples, a conformal or Weyl-type transformation of the fundamental tensor enables a description of dilatation associated, respectively, with cleavage surface roughness and nucleation of voids or vacancies. For the shear localization problem, the Finsler theory is able to accurately reproduce the surface energy of Griffith's fracture mechanics, and it predicts dilatation-induced toughening as observed in experiments on brittle crystals. For the cavitation problem, the Finsler theory is able to accurately reproduce the vacancy formation energy at a nanoscale resolution, and various solutions describe localized cavitation at the core of the body and/or distributed dilatation and softening associated with amorphization as observed in atomic simulations, with relative stability of solutions depending on the regularization length.

Keywords: Finsler geometry, continuum physics, elasticity, fracture, cavitation
MSC Codes: 51P05, 74A05, 74A45, 74B20
Subject Classifications: classical field theory (continuous media and variational approaches), differential geometry, geometric approaches to thermodynamics

1. Introduction

In Finsler geometry, each point on the base manifold can be envisioned as endowed with a vector of coordinates denoting its position from the origin and a director vector, also referred to herein as an internal state vector, whose components may or may not explicitly depend on position coordinates. Geometric objects such as metric tensors, connections, and derived quantities–e.g., torsion, curvature, and so forth–may in turn depend on both position and direction or internal state. This generality is in contrast to classical Riemannian geometry, wherein ultimate dependence of such geometric objects is on position alone. Finsler geometry encompasses certain geometries of Riemann,

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