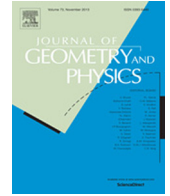




Contents lists available at ScienceDirect

## Journal of Geometry and Physics

journal homepage: [www.elsevier.com/locate/jgp](http://www.elsevier.com/locate/jgp)

## Differential invariants of self-dual conformal structures

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## ARTICLE INFO

## Article history:

Received 5 May 2016

Accepted 18 May 2016

Available online xxxx

## Keywords:

Differential invariants

Invariant derivations

Self-duality

Conformal metric structure

Hilbert polynomial

Poincaré function

## ABSTRACT

We compute the quotient of the self-duality equation for conformal metrics by the action of the diffeomorphism group. We also determine Hilbert polynomial, counting the number of independent scalar differential invariants depending on the jet-order, and the corresponding Poincaré function. We describe the field of rational differential invariants separating generic orbits of the diffeomorphism pseudogroup action, resolving the local recognition problem for self-dual conformal structures.

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## Introduction

Self-duality is an important phenomenon in four-dimensional differential geometry that has numerous applications in physics, twistor theory, analysis, topology and integrability theory. A pseudo-Riemannian metric  $g$  on an oriented four-dimensional manifold  $M$  determines the Hodge operator  $*$ :  $\Lambda^2 TM \rightarrow \Lambda^2 TM$  that satisfies the property  $*^2 = \mathbf{1}$  provided  $g$  has the Riemannian or split signature. In this paper we restrict to these two cases, ignoring the Lorentzian signature.

The Riemann curvature tensor splits into  $O(g)$ -irreducible pieces  $R_g = Sc_g + Ric_0 + W$ , where the last part is the Weyl tensor [1] and  $O(g)$  is the orthogonal group of  $g$ . In dimension 4, due to exceptional isomorphisms  $so(4) = so(3) \oplus so(3)$ ,  $so(2, 2) = so(1, 2) \oplus so(1, 2)$ , the last component splits further  $W = W_+ + W_-$ , where  $*W_{\pm} = \pm W_{\pm}$ . Metric  $g$  is called self-dual if  $*W = W$ , i.e.  $W_- = 0$ . This property does not depend on conformal rescalings of the metric  $g \rightarrow e^{2\phi}g$ , and so is the property of the conformal structure [g].

Since the space of  $W_-$  has dimension 5, and the conformal structure has 9 components in 4D, the self-duality equation appears as an underdetermined system of 5 PDE on 9 functions of 4 arguments. This is however a misleading count, since the equation is natural, and the diffeomorphism group acts as the symmetry group of the equation. Since  $\text{Diff}(M)$  is parametrized by 4 functions of 4 arguments, we expect to obtain a system of 5 PDE on  $5 = 9 - 4$  functions of 4 arguments.

This  $5 \times 5$  system is determined, but it has never been written explicitly. There are two approaches to eliminate the gauge freedom.

One way to fix the gauge is to pass to the quotient equation that is obtained as a system of differential relations (syzygies) on a generating set of differential invariants. By computing the latter for the self-dual conformal structures we write the quotient equation as a nonlinear  $9 \times 9$  PDE system, which is determined but complicated to investigate.

Another approach is to get a cross-section or a quasi-section to the orbits of the pseudogroup  $G = \text{Diff}_{\text{loc}}(M)$  action on the space  $\mathcal{SD} = \{[g] : W_- = 0\}$  of self-dual conformal metric structures. This was essentially done in the recent work [2, III.A]: By choosing a convenient ansatz the authors of that work encoded all self-dual structures via a  $3 \times 3$  PDE system

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