Physica D ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Fluctuations, response, and resonances in a simple atmospheric model

Andrey Gritsun [a,b,*], Valerio Lucarini [c,d,e]

[a] Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation
[b] Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russian Federation
[c] Department of Mathematics and Statistics, University of Reading, United Kingdom
[d] Centre for the Mathematics of the Planet Earth, University of Reading, United Kingdom
[e] CEN, University of Hamburg, Hamburg, Germany

HIGHLIGHTS

- We study the response of an atmospheric model to perturbations of its parameters.
- Methods of nonequilibrium statistical mechanics, Ruelle response theory are used.
- Response could be very different from the natural variability, the FDT is violated.
- Unexpected behavior for forcings with strong projections on the stable manifold.
- Resonant system behavior could be explained in terms of unstable periodic orbits.

ARTICLE INFO

Article history:
Received 26 April 2016
Received in revised form
22 February 2017
Accepted 27 February 2017
Available online xxxx
Communicated by H.A. Dijkstra

Keywords:
Atmospheric model
Resonant response
Covariant Lyapunov vectors
Ruelle response theory
Fluctuation–dissipation theorem
Unstable periodic orbits

ABSTRACT

We study the response of a simple quasi-geostrophic barotropic model of the atmosphere to various classes of perturbations affecting its forcing and its dissipation using the formalism of the Ruelle response theory. We investigate the geometry of such perturbations by constructing the covariant Lyapunov vectors of the unperturbed system and discover in one specific case – orographic forcing – a substantial projection of the forcing onto the stable directions of the flow. This results into a resonant response shaped as a Rossby-like wave that has no resemblance to the unforced variability in the same range of spatial and temporal scales. Such a climatic surprise corresponds to a violation of the fluctuation–dissipation theorem, in agreement with the basic tenets of nonequilibrium statistical mechanics. The resonance can be attributed to a specific group of rarely visited unstable periodic orbits of the unperturbed system. Our results reinforce the idea of using basic methods of nonequilibrium statistical mechanics and high-dimensional chaotic dynamical systems to approach the problem of understanding climate dynamics.

1. Introduction

Nowadays statistical mechanics and thermodynamics provide a comprehensive picture of the properties of equilibrium and near-equilibrium systems [1,2]. On the contrary, our understanding of nonequilibrium systems is comparatively poor and limited, despite the wealth of phenomena that are possible only far from equilibrium conditions. Advancing our knowledge on nonequilibrium systems is one of the great frontiers of contemporary science [3]. Conceptually, one can represent a large class of nonequilibrium systems as being in contact with at least two reservoirs having different temperature (and/or chemical potential), with a nonequilibrium steady state emerging after transients have died out [4]. The presence of nonequilibrium conditions is essential for sustaining, e.g., life or convective motions, through processes allowing for a systematic conversion of energy from one form to another one, like a heat engine. Additionally, irreversible processes lead to the generation of entropy, which is transferred into the reservoirs [5].

The climate is a prototypical example of a nonequilibrium, forced, and dissipative system, where the unequal absorption of solar radiation triggers a wealth of processes and feedbacks leading to the presence of a variability in the climatic fields covering a large range of scales in space and in time [6]. As a result, our ability to observe the climate state is intrinsically limited. It is

---

[*] Corresponding author at: Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation.

E-mail address: asgrit@mail.ru (A. Gritsun).

http://dx.doi.org/10.1016/j.physd.2017.02.015

extremely challenging to provide a convincing and comprehensive framework underpinning climate variability and able to predict its response to perturbations. This would be great importance for reconstructing the climate of the past and predicting its response to forcings such as changes in the atmospheric composition, land surface properties, incoming solar radiation, and position of the continents [7].

An extremely useful point of view on climate is given by thermodynamics: the organized motions of the geophysical fluids result from the transformation of available potential into kinetic energy, which is eventually dissipated by friction [7,8]. At this regard, one can construct a precise analogy between the functioning of the climate system and an imperfect, irreversible engine, characterized by the presence of positive correlations between heating patterns and temperature anomalies. In turn, the atmospheric winds and the oceanic currents contribute to decreasing the temperature and chemical potential (mostly due to inhomogeneity of water vapor – in the atmosphere – and salinity – in the ocean) gradients that fuel such motions: this provides stabilizing negative feedbacks that make it possible, together with the global Boltzmann radiative feedback, for the climate to reach a steady state [9].

In the case of (near) equilibrium systems, one can predict the impact of applying a (weak) forcing using the statistics of the unperturbed system through the fluctuation–dissipation theorem (FDT), which establishes a dictionary (in the form of linear operators) for translating natural into forced fluctuations, and vice versa [10,11]; extensions have been recently proposed for the nonlinear case [12].

Correspondingly, in order to bypass many of the bottlenecks mentioned above, it is very tempting to try to relate climate variability to its response to external forcings. In the case of Axiom A systems [13], it is indeed possible to construct a rigorous response theory, able to describe the change in the measure resulting from perturbing the system in terms of the properties of the unperturbed measure [14–16]. Nonetheless, in nonequilibrium conditions there is no obvious relationship between free and forced fluctuations of the system [17], as already suggested by Lorenz [18]. Physically, one can interpret such a lack of equivalence as resulting from the fact the natural fluctuations explore only the unstable manifold in the tangent space, while the forced fluctuations can explore both the stable and unstable directions. Mathematically, this has to do with the fact that the invariant measure is smooth only along the unstable directions of the flow. We remark that mathematical results strictly valid for Axiom A systems are considered extremely relevant for general high dimensional chaotic physical systems because of the chaotic hypothesis [19].

In order to bypass the problem of the lack of smoothness of the invariant measure and be able to take advantage of the FDT, various points of view have been proposed. In particular, some authors consider adding some stochastic forcing to the deterministic dynamics, so to consider the impact of unresolved scales [20]; see also discussion in [21,22]. This gives a rationale for using the FDT in the context of the climate models: while on one side there have been successful examples of applications of the FDT, predictive power depends substantially on the chosen observable [23–25].

The ab-initio implementation of the Ruelle response operators is hindered by the presence of differing behavior between the terms contributing along the stable and unstable directions of the flow [26]. Algorithms based on adjoint methods seem to partially ease these issues [27]. A possible idea for improving the convergence of the algorithm relies on projecting the perturbation flow on the unstable, neutral and stable covariant Lyapunov vectors [28] (CLVs), which have been recently constructed for rather nontrivial geophysical fluid dynamical systems [29,30]. Using a different approach, based on exploiting some formal properties and taking advantage of a set of test simulations, the Ruelle response theory has been shown to provide a considerable degree of predictive power in systems ranging from simple low dimensional models [31–33] to very high dimensional climate models with hundreds of thousands of degrees of freedom [9,34,35].

An alternative point of view on the problem can also be taken. In the case of Axiom A systems, an infinite set of unstable periodic orbits (UPOs) populate densely the attractor, and it is possible to evaluate the expectation value of a measurable observable as an average over the various UPOs, each taken with a suitable weight. The weights are smaller for more unstable orbits, so that the least unstable orbits give the largest contributions, and provide a natural way to reconstruct hierarchically the properties of the attractor [36–38]. While intuition seems to suggest that the numerics of constructing closed unstable orbits in chaotic systems (especially high-dimensional ones) is hopeless, this is indeed not the case: UPOs are used as practical tools for studying, interpreting, and reconstructing chaotic dynamical systems of various degrees of complexity, see [39].

UPOs have been constructed also in the case of simple yet relevant models of the atmosphere [40–42] and provide the framework of potentially addressing a classic problem of climate dynamics and dynamical meteorology, i.e. the identification of so-called modes or regimes, whose investigation has started through the classic concept of Grosswetterlagen [43]. More comprehensive approaches and definitions have been later given in the 1980s [44–46]; see an illuminating discussion in [47], while in [48] one can find many details on how to study and detect such regimes through time series analysis. The usual idea is that the least unstable UPOs might provide the key elements for reconstructing and understanding weather and climate regimes, and might be directly related to the principal modes of the system found using linear [48] and nonlinear [49] empirical methods, while transitions between the weather regimes could be interpreted as transitions between neighborhoods of UPOs [50]. This has crucial relevance in terms of performing extended prediction of the state of the climate system. We will address these specific research questions in a forthcoming publication.

The overall goal of this paper is to show the potential of some basic ideas of nonequilibrium statistical mechanics and high-dimensional chaotic dynamical systems to approach the problem of understanding climate dynamics. Using the mathematical framework of Axiom A dynamical system and taking advantage of the Ruelle response theory, we study the change in the statistical and dynamical properties of a simple quasi-geostrophic barotropic model [51,52] of the northern hemisphere atmosphere in a strongly chaotic regime, first described in [40], resulting from two different kinds of perturbations—namely, we shall change the orography, and the boundary layer dissipation of the system.

We anticipate some of the main results for the benefit of the reader. We find a relevant example of a case where a fundamental property on nonequilibrium system, i.e. the nonequivalence between forced and free motions, is apparent. We discover a resonance occurring in a frequency range where the unperturbed system does not have virtually any signal in terms of power spectrum. The resonance can be associated via UPOs analysis to an orographically forced Rossby-like wave, which corresponds to a set of extremely unstable UPOs. The analysis of CLVs shows that, indeed, a substantial part of the signal is associated to perturbed motions forced along the stable direction of the flow, so that they cannot be captured within the natural variability of the system. There is, in fact, no sign of such a Rossby-like wave within the variability of the unperturbed flow. Such a *climatic surprise* is a clear example in a high-dimensional nonequilibrium system of conditions under which the FDT can fail. Indeed, we show that the performance of the FDT in reconstructing the response of the system is much worse in the case of perturbations to the orographic forcing than in the case of perturbations to the

Please cite this article in press as: A. Gritsun, V. Lucarini, Fluctuations, response, and resonances in a simple atmospheric model, Physica D (2017), http://dx.doi.org/10.1016/j.physd.2017.02.015