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Semi-Global Persistence and Stability for a Class of Forced Discrete-Time Population Models

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Abstract

We consider persistence and stability properties for a class of forced discrete-time difference equations with three defining properties: the solution is constrained to evolve in the non-negative orthant, the forcing acts multiplicatively, and the dynamics are described by so-called Lur'e systems, containing both linear and non-linear terms. Many discrete-time biological models encountered in the literature may be expressed in the form of a Lur'e system and, in this context, the multiplicative forcing may correspond to harvesting, culling or time-varying (such as seasonal) vital rates or environmental conditions. Drawing upon techniques from systems and control theory, and assuming that the forcing is bounded, we provide conditions under which persistence occurs and, further, that a unique non-zero equilibrium is stable with respect to the forcing in a sense which is reminiscent of input-to-state stability, a concept well-known in nonlinear control theory. The theoretical results are illustrated with several examples. In particular, we discuss how our results relate to previous literature on stabilization of chaotic systems by so-called proportional feedback control.

Keywords: Absolute stability, control theory, density-dependent population models, forced systems, global asymptotic stability, input-to-state stability, Lur'e systems, population persistence
2010 MSC: 15B48, 37N25, 37N35, 39A22, 39A30, 93D09, 93D15

1. Introduction

We consider persistence and stability properties of forced discrete-time nonlinear models which take the form

$$x^\nabla = Ax + bh(c^T x, u)\,, \quad x(0) = x^0\,, \tag{1.1}$$

and which are constrained to evolve in the non-negative orthant of n-dimensional Euclidean space. Here x^∇ denotes the image of x under the left-shift operator, that is, $x^\nabla(t) = x(t+1)$ for all $t \in \mathbb{Z}_+$, where $\mathbb{Z}_+$ denotes the set of non-negative integers. Further, $x^0 \in \mathbb{R}^n_+$, $A \in \mathbb{R}^{n\times n}_+$, $b, c \in \mathbb{R}^n_+$ with c^T denoting the transpose of c, and $h : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a (nonlinear) function. The scalar-valued non-negative sequence u plays the role of a forcing term (sometimes termed and understood as a control or disturbance).

Models of the form (1.1) arise and have subsequently been studied in a variety of contexts, by a range of academic disciplines, and with a spectrum of techniques. The system (1.1) is an example of a positive system, and, in the special case that u is constant, it is a positive dynamical system, see [3, 28, 41, 42]. The state variables of positive systems are constrained to evolve in positive cones and, in the present context, the positive cone is simply $\mathbb{R}^n_+$, viewed as a subset of real n-dimensional Euclidean space, equipped with the usual partial ordering of componentwise inequality between vectors. Moreover, when

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