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M.D. Groves, D.J.B. Lloyd, A. Stylianou

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Pattern formation on the free surface of a ferrofluid: spatial dynamics and homoclinic bifurcation

M. D. Groves* D. J. B. Lloyd† A. Stylianou‡

Abstract

We establish the existence of spatially localised one-dimensional free surfaces of a ferrofluid near onset of the Rosensweig instability, assuming a general (nonlinear) magnetisation law. It is shown that the ferrohydrostatic equations can be derived from a variational principle that allows one to formulate them as an (infinite-dimensional) spatial Hamiltonian system in which the unbounded free-surface direction plays the role of time. A centre-manifold reduction technique converts the problem for small solutions near onset to an equivalent Hamiltonian system with finitely many degrees of freedom. Normal-form theory yields the existence of homoclinic solutions to the reduced system, which correspond to spatially localised solutions of the ferrohydrostatic equations.

1 Introduction

Ferrofluids

The Rosensweig instability - a surface instability of a ferrofluid - has been of interest since the 1960s (see Cowley & Rosensweig [6] and Rosensweig [22]). In an experiment, a vertical magnetic field is applied to a static ferrofluid layer, and regular cellular patterns (typically hexagons) emerge on the fluid surface as the field strength is increased through a critical value (see Gollwitzer *et al.* [10, 11] for recent experimental results). Recently, experiments have shown that spatially localised free-surface structures occur in the hysteresis region between the flat state and the cellular spatially periodic patterns (see Figure 1, Richter [20], Richter & Barashenkov [21] and Lloyd *et al.* [17] for experimental results, and Lavrova *et al.* [16] and Cao & Ding [5] for finite-element simulations). Despite this wealth of experimental and numerical evidence little is known theoretically on the existence of localised solutions to the ferrohydrostatic equations.

*Fachrichtung Mathematik, Universität des Saarlandes, Postfach 151150, 66041 Saarbrücken, Germany; Department of Mathematical Sciences, Loughborough University, Loughborough, LE11 3TU, UK

†Department of Mathematics, University of Surrey, Guildford, GU2 7XH, UK

‡Institut für Mathematik, Universität Kassel, 34132 Kassel, Germany