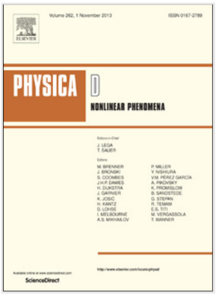

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PII: S0167-2789(16)30481-X
DOI: http://dx.doi.org/10.1016/j.physd.2017.02.003
Reference: PHYSD 31885

To appear in: *Physica D*

Received date: 23 September 2016
Accepted date: 7 February 2017

Please cite this article as: A. Bayliss, V.A. Volpert, Complex predator invasion waves in a Holling-Tanner model with nonlocal prey interaction, *Physica D* (2017), http://dx.doi.org/10.1016/j.physd.2017.02.003

Complex Predator Invasion Waves in a Holling-Tanner Model with Nonlocal Prey Interaction

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Abstract

We consider predator invasions for the nonlocal Holling-Tanner model. Predators are introduced in a small region adjacent to an extensive predator-free region. In its simplest form an invasion front propagates into the predator-free region with a predator-prey coexistence state displacing the predator-free state. However, patterns may form in the wake of the invasion front due to instability of the coexistence state. The coexistence state can be subject to either oscillatory or cellular instability, depending on parameters. Furthermore, the oscillatory instability can be either at zero wave number or finite wave number. In addition, the (unstable) predator-free state can be subject to additional cellular instabilities when the extent of the nonlocality is sufficiently large. We perform numerical simulations that demonstrate that the invasion wave may have a complex structure in which different spatial regions exhibit qualitatively different behaviors. These regions are separated by relatively narrow transition regions that we refer to as fronts. We also derive analytic approximations for the speeds of the fronts and find qualitative and quantitative agreement with the results of computations.

1. Introduction

In this paper we consider the Holling-Tanner model [1] for a predator-prey system, accounting for diffusion of both species as well as nonlocal prey intraspecies competition. Our focus is the predator invasion problem in which predators are introduced into a predator-free state via a small coexistence region. An invasion wave is generated and propagates into the predator-free region displacing the predator-free state by a predator-prey coexistence state. In its simplest form, this wave can be thought of as a stable state (coexistence) displacing an unstable state (predator-free). However, this scenario is valid only for a restricted parameter regime, as the coexistence state can be subject to both oscillatory and cellular instabilities depending on parameters. Furthermore, the unstable predator-free state can be subject to an additional cellular instability for sufficiently extensive nonlocality. As a result, spatiotemporal patterns can occur in the wake of the invasion wave as well as in the predator-free state. Thus, the invasion wave may have a complex structure in which different spatial regions exhibit qualitatively different behaviors. These regions are separated by relatively narrow transition regions that we refer to as "fronts". We use the term front in a descriptive sense, it does not refer, for example, to a discontinuity in the solution or its derivatives or to a heteroclinic orbit connecting two equlibrium states. Specifically we use the term "invasion front" to describe the transition region between the coexistence region and the predator-free region into which the coexistence regime propagates. As an illustration of such structures refer to the space-time visualization of one such invasion pattern in Figure 5a below. Here the coexistence region consists of two distinct regions, a region where the solution is a constant in space and time followed by a region with a wave train propagating to the left (away from the constant region). We also use the term front to describe the transition region between different behaviors within the coexistence region (in the context of Figure 5a the transition region separating the constant coexistence state from the wave train).

Many of the patterns involve wave trains propagating away from the invasion front. These wave trains need not be strict traveling waves, as shown below. There is some field evidence for such behavior, e.g., [2], although the precise mechanisms generating the observed waves are not clear and the data analysis of some observations is not sufficient to fully categorize the dynamics and spatial structure of the waves.

It has long been recognized that competition (both intraspecies and interspecies) is often nonlocal. Nonlocal

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