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Abstract

It is shown that van Iterson's metric for disk packings, proposed in 1907 in the study of a centric model of spiral phyllotaxis, defines a bounded distance function in the plane. This metric is also related to the bifurcation of Voronoi tilings for logarithmic spiral lattices, through the continued fraction expansion of the divergence angle. The phase diagrams of disk packings and Voronoi tilings for logarithmic spirals are dual graphs to each other. This gives a rigorous proof that van Iterson's diagram in the centric model is connected and simply connected. It is a nonlinear analog of the duality between the phase diagrams for disk packings and Voronoi tilings on the linear lattices, having the modular group symmetry.

Keywords: spiral phyllotaxis, disk packing, Voronoi tiling, continued fraction
2000 MSC: 11A55, 52C15, 52C20, 92C80

1. Introduction

Phyllotaxis is the study on the distribution of leaves, florets, and seeds around an axis. A historical overview is in [2], and an extensive review on recent developments is given in [13]. Among the classical works in geometric approach, van Iterson [17] studied, in 1907, the packing of disks and the shape called 'folioids', in the cylinder, the plane (called the centric model) and the cone. Since his work (see Figure 1), the phase diagrams of disk packings with the Farey tree structure have been reproduced by other researchers under various models and hypotheses of phyllotaxis [6, 11, 12].

The cylindrical model usually considers the linear lattices, and its geometry is well established. The parameter space has the modular group($PSL(2,\mathbb{Z})$) symmetry [3]. In the cylindrical model, it is known that the phase diagram of disk packings is dual to the phase diagram of Voronoi tilings [9].

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