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Low-Dimensional Reduced-Order Models for Statistical Response and Uncertainty Quantification: Barotropic Turbulence with Topography

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Abstract

A low-dimensional reduced-order statistical closure model is developed for quantifying the uncertainty to changes in forcing in a barotropic turbulent system with topography involving interactions between small-scale motions and a large-scale mean flow. Imperfect model sensitivity is improved through a recent mathematical strategy for calibrating model errors in a training phase, where information theory and linear statistical response theory are combined in a systematic fashion to achieve the optimal model parameters. Statistical theories about a Gaussian invariant measure and the exact statistical energy equations are also developed for the truncated barotropic equations that can be used to improve the imperfect model prediction skill. A stringent paradigm model of 57 degrees of freedom is used to display the feasibility of the reduced-order methods. This simple model creates large-scale zonal mean flow shifting directions from westward to eastward jets with an abrupt change in amplitude when perturbations are applied, and prototype blocked and unblocked patterns can be generated in this simple model similar to the real natural system. Principal statistical responses in mean and variance can be captured by the reduced-order models with desirable accuracy and efficiency with only 3 resolved modes. An even more challenging regime with non-Gaussian equilibrium statistics using the fluctuation equations is also tested in the reduced-order models with accurate prediction using the first 5 resolved modes. These reduced-order models also show potential for uncertainty quantification and prediction in more complex realistic geophysical turbulent dynamical systems.

Keywords: Reduced-order methods, topographic barotropic system, statistical responses, uncertainty quantification

1. Introduction

Turbulent dynamical systems characterized by a large dimensional phase space and many degrees of strong instabilities transferring energy throughout the system are ubiquitous in science and engineering [1, 2, 3, 4]. Situations of obvious importance in atmosphere and ocean science occur when smaller-scale motions have a significant feedback and interaction with a larger-scale mean flow [5, 6, 7]. The feedback and interaction induce instability that can make the system very sensitive to even small changes in external forcing perturbations. One prototype situation of this sort occurs in the interaction of large-scale and small-scale components of barotropic flow over topography via topographic stress. The influence of large-scale anisotropic topographic variations on the fluid forms alternative blocked and unblocked states relative to the model sensitivity to the forcing. The simplest set of equations that meaningfully describes the motion of the large-scale geophysical flows is given by the quasi-geostrophic barotropic equations [3, 4]. They are the result of "filtering out" the fast gravity waves from the rotating barotropic equations. The multiple equilibrium states of this barotropic system with dissipation and single-mode topography are studied in [8, 9], suggesting the possible importance as model states of atmospheric blocking and changes due to variations in external forcing. The system can also be extended to a number of important climate models directly such as two-layer models or barotropic flow on the sphere, characterizing a wider category of realistic phenomena from nature.

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