Accepted Manuscript

Microorganism billiards

Saverio E. Spagnolie, Colin Wahl, Joseph Lukasik, Jean-Luc Thiffeault

PII: S0167-2789(15)30053-1
DOI: http://dx.doi.org/10.1016/j.physd.2016.09.010
Reference: PHYSD 31855

To appear in: *Physica D*

Received date: 17 July 2015
Accepted date: 29 September 2016

Please cite this article as: S.E. Spagnolie, C. Wahl, J. Lukasik, J.-L. Thiffeault, Microorganism billiards, *Physica D* (2016), http://dx.doi.org/10.1016/j.physd.2016.09.010

Microorganism Billiards

Saverio E. Spagnolie, Colin Wahl, Joseph Lukasik and Jean-Luc Thiffeault

Department of Mathematics, University of Wisconsin – Madison
480 Lincoln Dr., Madison, WI 53706

Abstract

Recent experiments and numerical simulations have shown that certain types of microorganisms "reflect" off of a flat surface at a critical angle of departure, independent of the angle of incidence. The nature of the reflection may be active (cell and flagellar contact with the surface) or passive (hydrodynamic) interactions. We explore the billiard-like motion of a body with this empirical reflection law inside a regular polygon and show that the dynamics can settle on a stable periodic orbit or can be chaotic, depending on the swimmer's departure angle and the domain geometry. The dynamics are often found to be robust to the introduction of weak random fluctuations. The Lyapunov exponent of swimmer trajectories can be positive or negative, can have extremal values, and can have discontinuities depending on the degree of the polygon. A passive sorting device is proposed that traps swimmers of different departure angles into separate bins. We also study the external problem of a microorganism swimming in a patterned environment of square obstacles, where the departure angle dictates the possibility of trapping or diffusive trajectories.

Keywords: Billiards, Non-specular reflections, Microorganism locomotion, Sorting

1. Introduction

Microorganisms often follow unexpected trajectories when swimming near surfaces. Organisms such as spermatozoa and *E. coli* are attracted to surfaces