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Numerical analysis of the rescaling method for parabolic problems with blow-up in finite time

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Abstract

In this work, we study the numerical solution for parabolic equations whose solutions have a common property of blowing up in finite time and the equations are invariant under the following scaling transformation

$$u \mapsto u_\lambda(x,t) := \lambda^{\frac{2}{p-1}} u(\lambda x, \lambda^2 t).$$

For that purpose, we apply the rescaling method proposed by Berger and Kohn [9] to such problems. The convergence of the method is proved under some regularity assumption. Some numerical experiments are given to derive the blow-up profile verifying henceforth the theoretical results.

Keywords: Numerical blow-up, finite-time blow-up, nonlinear parabolic equations.

1. Introduction

We study the solution of the following parabolic problem

$$\begin{cases} u_t(x,t) &= u_{xx}(x,t) + g(u,u_x), & \text{in} \quad \Omega \times (0,T), \\ u(x,t) &= 0 & \text{on} \quad \partial\Omega \times [0,T), \\ u(x,0) &= u_0(x), & \text{on} \quad \bar{\Omega}. \end{cases} \tag{1}$$

where $u(t) : x \in \Omega \mapsto u(x,t) \in \mathbb{R}$, $p > 1$. The function g is given by

$$g(u,u_x) = |u|^{p-1}u + \beta |u_x|^q, \quad \text{with} \quad q = \frac{2p}{p+1},$$

for some $\beta \in \mathbb{R}$. This equation can be viewed as a population dynamic model (see [46] for an example).
We also consider the complex Ginzburg-Landau equation,

$$\begin{cases} u_t(x,t) &= (1+\imath\gamma)u_{xx} + (1+\imath\delta)|u|^{p-1}u, & \text{in} \quad \Omega \times (0,T), \\ u(x,t) &= 0 & \text{on} \quad \partial\Omega \times [0,T), \\ u(x,0) &= u_0(x), & \text{on} \quad \bar{\Omega}. \end{cases} \tag{2}$$

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