ALGEBRAIC RANDOM WALKS IN THE SETTING OF SYMMETRIC FUNCTIONS

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Using the standard formulation of algebraic random walks (ARWs) via coalgebras, we consider ARWs for co- and Hopf-algebraic structures in the ring of symmetric functions. These derive from different types of products by dualisation, giving the dual pairs of outer multiplication and outer coproduct, inner multiplication and inner coproduct, and symmetric function plethysm and plethystic coproduct. Adopting standard coordinates for a class of measures (and corresponding distribution functions) to guarantee positivity and correct normalisation, we show the effect of appropriate walker steps of the outer, inner and plethystic ARWs. If the coordinates are interpreted as heights or occupancies of walker(s) at different locations, these walks introduce translations, dilations (scalings) and inflations of the height coordinates, respectively.

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1. Introduction and motivation

The subject of algebraic random walks (ARWs) is a development of recent interest in the subject of ‘quantum probability’ [1, 2]. The latter has origins in the foundations of quantum mechanics, but more recently has been influenced by trends towards noncommutative probability. ARWs use a very general algebraic framework [3–5], in formulating a variety of stochastic processes such as Markov and Lévy processes [6], via a Hopf algebraic formulation [7]. Early studies used commutative function algebras and q-extensions to study Brownian motion random walks [8, 9]; and more recently ARWs have been formulated for a variety of other (co)algebraic structures such as the Weyl–Heisenberg algebra and q-deformations thereof [10], as well as Lie algebraic enveloping algebras [11, 12], and anyonic algebras. Interest in these studies is in the relationship of asymptotic limits to known (classical) stochastic processes, and also to some new types which have been shown to have behaviour which is significantly nonclassical, such as variances linear in the step number rather than its square root, for example. For references and recent work see [11, 13–15]. For a review of the related topic of quantum random walks we refer to Konno [16].

The purpose of this work is to exploit the universal symmetric function ring $\Lambda(X)$ as an arena for ARWs. As is well known, $\Lambda(X)$ can be formulated as a Hopf algebra [17, 18] due to its intimate connection with character theory for the classical groups [19]. In fact, several algebraic and co-algebraic structures coexist [20, 21] which means that there are rich possibilities for defining different types of ARWs. Specifically, we shall consider the outer Hopf algebra, the inner coalgebra [20] and a plethysm coalgebra [21, 22]; these are all naturally defined by dualising standard products or pairings, namely the outer and inner symmetric function products, and the plethysm product, respectively (for notation see below); all can potentially lead to ARWs or composite variations on ARWs, and this will turn out to be the case.

The main point of comparison for the symmetric function algebraic random walks which we shall describe, will be that of a random walk on a line. This is to be viewed in a discretized limit as in its classical ARW treatment by Majid [8, 9]. We will have discrete walker locations (in this case belonging to $\mathbb{N}$), and walk steps implemented by rearranging 'height' coordinates representing the walker occupancy of the locations. We shall see that the different symmetric function coproducts involved in the convolutions operative in the time steps, lead to different classes of evolution. The main purpose of this paper is to point out that these may be significantly richer than the standard case. Not only can local *translations* (additive changes to heights), but also local *scalings* (multiplicative changes to the heights) be generated; there are also global *inflations* of the sequence of height coordinates (for example, decimation, whereby heights are reallocated at intervals of 10 units, leaving intervening zeroes). In the present note we do not consider these new possibilities in depth, but restrict ourselves simply to an identification or enumeration of the various cases of symmetric function ARWs (consistent with constraints such as positivity and normalisation). We defer investigations of asymptotic behaviour, continuum limits such as master or diffusion equations and the like, as well as interesting questions like identifying stationary states of some of the nonstandard evolutions, to later detailed studies.

We emphasize that while our current work has not developed such applications, nonetheless the setting of symmetric functions is a combinatorially central and rich context for investigation of the ARW theme. Further discussion along these lines, on the interpretation of our findings, and some comparisons with other approaches into probability measures and Markov processes on symmetric functions, is given in the concluding remarks below. To complete this introductory section, we briefly review the technical framework of the ARW formalism [7–9].

1.1. The ARW formalism

We work in the algebraic framework as developed especially via the study of quantum probability [3], although our current application to standard symmetric

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