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TRANSITION OPERATORS ASSIGNED TO PHYSICAL SYSTEMS

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By a physical system we recognize a set of propositions about a given system with their truth values depending on the states of the system. Since every physical system can go from one state to another one, there exists a binary relation on the set of states describing this transition. Our aim is to assign to every such system an operator on the set of propositions which is fully determined by the mentioned relation. We establish conditions under which the given relation can be recovered by means of this transition operator.

Keywords: physical system, transition relation, transition operators, states, complete lattice, transition frame.

1. Introduction

In 1900, D. Hilbert formulated his famous 23 problems, see e.g. [8] for details. In problem number 6, he asked: “Can physics be axiomatized?” This means that he asked if physics can be formalized and/or axiomatized to reach a logically perfect system forming a basis of precise physical reasoning. This challenge was followed by G. Birkhoff and J. von Neuman in 1930s producing the so-called logic of quantum mechanics. We are going to adopt the method and examples of D. J. Foulis from [6] and [7], however, we are not restricted to the logic of quantum mechanics. We are focused on a general situation with a physical system endowed with states which it can reach. Our goal is to assign to every such system $\mathcal{P}$ a certain logic **B** (formulated by an observer) informing on the system relative to some set of states

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of $\mathcal{P}$ and a so-called transition operator T that aims to express a dynamics of this logic $\mathbf{B}$ which would be in accordance with the passage of the system $\mathcal{P}$ from one state to another one. Conditions under which this assignment works perfectly will be formulated.

We start with a formalization of a given physical system. It is assumed that generally a physical system may be in a number of states and that these states can be determined by the measurements of observables. Thus every state of the physical system is the result of a preparation of the system. After the measurement of an observable, the system will be in a state that has been prepared by this measurement. From the logical point of view, we can formulate propositions saying what has been measured on the physical system, or what preparations have been performed.

Denote by S the set of states of a given physical system $\mathcal{P}$. It is given by the nature of $\mathcal{P}$ from what state $s \in S$ the system $\mathcal{P}$ can move to a state $t \in S$. Hence, there exists a binary relation R on S such that $(s,t) \in R$. This process is called a *transition* of $\mathcal{P}$ and R is called a *transition relation* of $\mathcal{P}$.

In addition to the previous, the observer of $\mathcal{P}$ can formulate propositions revealing our knowledge about the system with respect to the transition. The truth values of these propositions depend on states. For example, the proposition p can be true if the system $\mathcal{P}$ is in the state s_1 but false if $\mathcal{P}$ is in the state s_2. Hence, for each state $s \in S$ we can evaluate the truth value of p, which is denoted by $p(s)$. The set of all truth values for all propositions will be called the *table*.

Now, denote by B the set of propositions about the physical system $\mathcal{P}$ formulated by the observer. We can introduce a partial order $\leq$ on B as follows:

$$\text{for } p, q \in B,\ p \leq q \text{ if and only if } p(s) \leq q(s) \text{ for all } s \in S.$$

One can immediately check that the *contradiction*, i.e. the proposition with constant truth value 0, is the least element and the *tautology*, i.e. the proposition with the constant truth value 1 is the greatest element of the partially ordered set $(B; \leq)$; this fact will be expressed by the notation $\mathbf{B} = (B; \leq, 0, 1)$ for the bounded partially ordered set of propositions about $\mathcal{P}$. This partially ordered set $\mathbf{B} = (B; \leq, 0, 1)$ will be referred to as a *logic of* $\mathcal{P}$.

However, our physical system $\mathcal{P}$ is dynamical which is captured by its transition and it is described by the transition relation R. Our aim is to set up a dynamic logic based on $(B; \leq, 0, 1)$ which can formalize the process of transition.

Assume that we are given a partially ordered set $\mathbf{M} = (M; \leq, 0, 1)$ which will play the role of the set of truth values of our logic. Then we will assign to $(B; \leq, 0, 1)$ an operator $T\colon B \to M^S$. This means that T transforms every proposition from B into a sequence of truth values (into a sequence of 0's and 1's if our logic is two-valued). This sequence can be considered again as a proposition on $\mathcal{P}$ depending on states from S but it need not belong to B in general. Such an operator will be called a *transition operator* of $(B; \leq, 0, 1)$ if it is in accordance with the transition relation R in the sense which will be given later. Then the formal system $(B; \leq, 0, 1, T)$ will be considered as a *dynamic logic* describing the behaviour of our physical system $\mathcal{P}$.

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