

BOUNDARY CONDITIONS FOR INFINITE CONSERVATION LAWS

V. ROSENHAUS

Department of Mathematics and Statistics, California State University Chico, Chico, CA 95929, USA
(e-mail: vrosenhaus@csuchico.edu)

and

M. S. BRUZÓN and M. L. GANDARIAS

Departamento de Matemáticas, Universidad de Cádiz, P.O.Box 40 11510 Puerto Real, Cádiz, Spain
(e-mails: m.bruzon@uca.es, marialuz.gandarias@uca.es)

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Regular soliton equations (KdV, sine-Gordon, NLS) are known to possess infinite sets of local conservation laws. Some other classes of nonlinear PDE possess infinite-dimensional symmetries parametrized by arbitrary functions of independent or dependent variables; among them are Zabolotskaya–Khokhlov, Kadomtsev–Petviashvili, Davey–Stewartson equations and Born-Infeld equation. Boundary conditions were shown to play an important role for the existence of local conservation laws associated with infinite-dimensional symmetries. In this paper, we analyze boundary conditions for the infinite conserved densities of regular soliton equations: KdV, potential KdV, Sine-Gordon equation, and nonlinear Schrödinger equation, and compare them with boundary conditions for the conserved densities obtained from infinite-dimensional symmetries with arbitrary functions of independent and dependent variables.

Keywords: conservation laws; symmetries; boundary conditions.

1. Introduction

It is known that soliton equations (solvable by inverse scattering method) possess infinite series of local conservation laws, see e.g. [2]. In this paper, we attempt to compare these infinite series of conservation laws with sets of conservation laws that follow from the existence of infinite-dimensional (infinite) symmetry groups (pseudo-groups) for certain classes of nonlinear PDE's. In particular, we discuss nonlinear equations that admit infinite symmetry groups parametrized by arbitrary functions of independent as well as dependent variables, and the corresponding sets of local conservation laws, see e.g. [28–31]. In the paper, we aim to compare boundary (asymptotic) conditions responsible for the existence of the sets of local conserved densities. This work could contribute to a better understanding of the factors determining integrability of a nonlinear system.

According to the second Noether theorem [21] (see also [23]), infinite variational symmetries with arbitrary functions of all independent variables lead, instead of

nontrivial conservation laws, to certain identity relations between equations of the original differential system and their derivatives (meaning that the original differential system is underdetermined). Infinite variational symmetries with arbitrary functions of not all independent variables were studied in [28], and were shown to lead to a finite number of essential (integral) local conservation laws. Arbitrary functions of spatial variables give rise to just one essential conservation law (conserved density), and arbitrary functions of time lead to a finite number of essential conservation laws. Each essential conservation law was shown to be determined by a specific form of boundary conditions, see e.g. [28–30, 33]. The situation with infinite symmetry algebras parametrized by arbitrary functions of dependent variables is radically different, leading to an infinite set of essential conservation laws, and the corresponding boundary conditions (at infinity) in this case turn out to be quite soft (standard) [31, 32, 34]. Two known examples of this situation are for equations of Liouville type (see [40, 41]) that can be integrated by the Darboux method, e.g. [8, 36], and hydrodynamic-type equations [6, 35].

In the present paper, we analyze boundary conditions for the infinite conserved densities of known soliton equations: KdV, potential KdV, sine-Gordon equation, and nonlinear Schrödinger equation, and compare them with boundary conditions for the conserved densities obtained from infinite symmetries with arbitrary functions of independent and dependent variables.

2. Infinite symmetries and essential conservation laws

Let us briefly outline the approach we follow; for details see [28] and [31]. By a conservation law for a differential system

$$\begin{aligned} \omega^a(x, u, u_i, \dots) &= 0, & i &= 1, \dots, m+1, & a &= 1, \dots, n, \\ u_i^a &\equiv \partial u^a / \partial x^i, & x^i &= (x^1, x^2, \dots, x^m, t), \end{aligned}$$

we mean a continuity equation

$$D_i K_i \doteq 0, \quad K_i = K_i(x_j, u^a, u_j \dots), \quad i, j = 1, \dots, m+1, \quad a = 1, \dots, n,$$

(K_i are smooth functions) which is satisfied for any solutions of the original system. Each conservation law is defined up to an equivalence transformation $K_i \rightarrow K_i + P_i$, $D_i P_i \doteq 0$, see [23]. Two conservation laws belong to the same equivalence class if they differ by a trivial conservation law. A conservation law $D_i K_i \doteq 0$ is trivial if vector K_i vanishes on solutions: $K_i \doteq 0$ ($i = 1, \dots, m+1$), if the continuity equation is satisfied in the whole space: $D_i K_i = 0$ [23], or if a linear combination of these two factors is taking place.

By an **essential** conservation law [28], we mean such nontrivial conservation law $D_i K_i \doteq 0$, which gives rise to a nonvanishing conserved quantity

$$D_t \int_D K_t dx^1 dx^2 \cdots dx^m \doteq 0, \quad x \in D \subset R^{m+1}, \quad K_t \not\equiv 0. \quad (2.1)$$

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