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Negative group velocity in solids

Kert Tamm *, Tanel Peets, Jüri Engelbrecht, Dmitri Kartofelev
Centre for Nonlinear Studies, Institute of Cybernetics at Tallinn University of Technology, Akadeemia Tee 21, 12618 Tallinn, Estonia

HIGHLIGHTS

- Dispersive waves in solids with hierarchical microstructure are studied.
- Dispersion analysis shows pre-resonant states resulting in negative group velocity.
- The influence of material parameters on the evolution of wave profiles is shown.
- The negative group velocity can lead to a smaller "effective" dispersion.

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ABSTRACT

Waves with the negative group velocity (NGV) are known to exist in optics (Sommerfeld and Brillouin) and in some mechanical cases like layered media, cylindrical shells and cylinders. In this paper the effects of the NGV on the evolution of the wave profiles are studied in the context of a Mindlin type continuum model with two microstructures in the 1D setting. Based on dispersion analysis, the range of parameters when the NGV region exists is determined. Numerical analysis is used to establish effects of the NGV in the evolution of wave profiles in time. The results can be used in material science.

1. Introduction

The negative group velocity (NGV) is an interesting phenomenon usually attributed to optics [1–3]. As far as this phenomenon is related to wave propagation, it is not surprising that the NGV can also exist for deformation waves in solids. It was shown already by H. Lamb [4] for transverse vibrations of strings even earlier than famous studies in optics [3]. In physical terms, the NGV appears for Lamb waves in layered media (solid–liquid–solid) [5], for plates both experimentally (see [6,7] and references therein) and theoretically (see [8,9] and references therein), for waves in cylindrical shells [10] or cylinders [11], for waves in metamaterials [12,13], etc. We noticed the appearance of the NGV for longitudinal waves in microstructured materials with multiple scales (a scale within a scale) [14,15]. In this case the dispersion analysis shows the existence of three dispersion curves: one acoustic branch and two optical branches. For some sets of material parameters two optical branches are close to each other. As far as optical branches describe non-propagating oscillations, it was conjectured in [15] that at such a pre-resonant situation these non-propagating oscillations are coupled resulting in the NGV. Clearly further studies are needed for understanding this interesting phenomenon in order to establish the dependence of the NGV on physical parameters of the microstructure and the influence of the NGV on wave profiles. The latter effect is interesting because in optics usually the NGV is space-dependent, but the NGV in microstructured solids depends on wavenumbers (frequencies).

* Corresponding author.
E-mail addresses: kert@ioc.ee (K. Tamm), tanelp@ioc.ee (T. Peets), je@ioc.ee (J. Engelbrecht), dima@ioc.ee (D. Kartofelev).

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In this paper further analysis is presented for Mindlin-type models describing the microstructured solids [14,16–18]. The attention is focused (i) to establishing the regions of parameters where the NGV can exist and (ii) to describing the changes of wave profiles in regions where the NGV exists. In Section 2 the governing equations are presented together with sets of material parameters used in the further analysis. Section 3 is devoted to the dispersion analysis. The detailed study of group and phase velocities permits to reveal the changes in dispersion characteristics due to changes in material parameters and establish the basis for numerical analysis. In Section 4 the ideas of the pseudospectral method used in numerics are described. The main results of the analysis are presented in Section 5 while in Section 6 final remarks are given.

2. Governing equations

In the present paper a mathematical model for microstructured solids is considered which can have the NGV regions in their dispersion curves under some parameter combinations. The derivation of the governing equations is briefly the following. We start with Lagrangian $L = K - W$, where K is the kinetic and W is the potential energy and derive the governing equations by using Euler–Lagrange equations after determining the K and W. For two microstructures with different scales one of the simplest potentials W which accounts for nonlinear and dispersive terms can be taken as (see [19,20] and references therein)

$$W = \frac{Y}{2}u_x^2 + A_1\varphi_1 u_x + \frac{B_1}{2}\varphi_1^2 + \frac{C_1}{2}\varphi_{1x}^2 + A_{12}\varphi_{1x}\varphi_2 + \frac{B_2}{2}\varphi_2^2 + \frac{C_2}{2}\varphi_{2x}^2 + A_2\varphi_2 u_x + \frac{N}{6}u_x^3 + \frac{M_1}{6}\varphi_{1x}^3 + \frac{M_2}{6}\varphi_{2x}^3, \tag{1}$$

where u is the macrodisplacement, φ_i are microdeformations and capital letters denote material coefficients. Subscript x denotes the spatial, and t the time derivative, respectively. If we take $A_{12} = 0$ then we get a double microstructure model where concurrent microstructures do not interact. Taking $A_2 = 0$, results in a hierarchical microstructure model where the second microstructure is embedded into the first one. In the following we deal with the case $A_2 = 0$. Following the Euler–Lagrange formalism (see [14,15,20] for details), the system of governing equations in the dimensionless normalised form is

$$\begin{aligned}
U_{TT} &= U_{XX} + \alpha_1\Phi_{1X} + \alpha_2\Phi_{2X} + \alpha_3 U_X U_{XX},\\
\Phi_{1TT} &= \beta_1\Phi_{1XX} + \beta_2\Phi_{2X} - \beta_3\Phi_1 + \beta_4\Phi_{1X}\Phi_{1XX} - \beta_5 U_X,\\
\Phi_{2TT} &= \zeta_1\Phi_{2XX} - \zeta_2\Phi_{1X} - \zeta_3\Phi_2 + \zeta_4\Phi_{2X}\Phi_{2XX} - \zeta_5 U_X,
\end{aligned} \tag{2}$$

where coefficients in terms of material and geometrical parameters are expressed as

$$\begin{aligned}
&\alpha_1 = \frac{A_1L^2}{l_1U_oY}, \quad \alpha_2 = \frac{A_2L^2}{l_2U_oY}, \quad \alpha_3 = \frac{NU_o}{LY},\\
&\beta_1 = \frac{C_1\rho}{I_1Y}, \quad \beta_2 = \frac{A_{12}l_1L\rho}{I_1l_2Y}, \quad \beta_3 = \frac{B_1L^2\rho}{I_1Y}, \quad \beta_4 = \frac{M_1\rho}{I_1l_1Y}, \quad \beta_5 = \frac{A_1l_1U_o\rho}{I_1Y},\\
&\zeta_1 = \frac{C_2\rho}{I_2Y}, \quad \zeta_2 = \frac{A_{12}l_2L\rho}{I_2l_1Y}, \quad \zeta_3 = \frac{B_2L^2\rho}{I_2Y}, \quad \zeta_4 = \frac{M_2\rho}{I_2l_2Y}, \quad \zeta_5 = \frac{A_2l_2U_o\rho}{I_2Y}.
\end{aligned}$$

Here ρ is the density, I_i are the microinertia, l_i are the characteristic scales of the microstructures ($i = 1, 2$), U_0 is the amplitude and L is the wavelength of the initial excitation. For the sake of clarity it should be noted that the change of variables for the dimensionless form is $x = LX$, $t = (LT)\sqrt{\rho/Y}$, $u = U_oU$, $\phi_i = (L/l_i)\Phi_i$ and the ratio L/l_i has been introduced to take into account the scale separation between microstructures explicitly. The microdeformations are dimensionless to start with and the introduced ratio maintains that property (see [14] for details).

Based on earlier research [15], we note that model (2) leads to three dispersion curves — one acoustic and two optical branches. We pick up the material parameters in a way that results in three different cases for the NGV. The first case is where there is no NGV region, the second case is when the acoustic branch has a local minimum at zero group velocity at a certain wavenumber and the last case is when there is a NGV region at a certain range of wavenumbers in the acoustic branch. In addition, as a result of normalising the equations some additional constraints have been introduced for the material parameters resulting in a situation where the first optical branch starts from the dimensionless frequency equal to one at low wavenumbers (i.e., $\xi < 1$) and the second optical branch starts from the dimensionless frequency equal to two. For the normalisation the dimensionless speed of sound for the bulk medium has been taken equal to one. The NGV condition is controlled by changing parameters A_{12} and C_1. Parameter B_2 is kept constant.

For the potential W, (relation (1)) the chosen parameters for most calculations are:
$Y = 100,\ A_1 = 5,\ A_2 = 0,\ B_1 = 10,\ B_2 = 16,\ C_2 = 8,\ N = M_1 = M_2 = 0,$
while material and geometrical parameters are:
$\rho = 100,\ I_1 = 10,\ I_2 = 4,\ U_o = 1,\ L = 1,\ l_1 = \frac{1}{4},\ l_2 = \frac{1}{20}.$

The parameters A_{12} and C_1 are given in Table 1. The "Zero NGV" case in Table 1 is considered the reference case meaning that if C_1 is varied then parameter A_{12} is kept at its reference value and vice versa. Later also the case $N \neq 0,\ M_1 \neq 0$ and $M_2 \neq 0$ is analysed.

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