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Wave generation and source energy distribution in cylindrical fluid-filled waveguide structures

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HIGHLIGHTS

- Explicit expressions for bulk, tube and leaky waves generated by an in-hole source.
- Leaky wave asymptotics valid for the whole range of propagation directions.
- Emergence of new guided waves due to a fluid saturation of porous interlayers.
- Source energy partition among the generated tube, bulk and leaky waves.

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ABSTRACT

The paper deals with an analytically based computer simulation of wave fields generated by a monopole source in a multilayered fluid-filled tube waveguide in an elastic or porous-elastic space. The mathematical problems considered relate to various practical applications ranging from a micro scale for fibers and nano-tubes to a macro scale for seismo-acoustics and well production. To be specific, the presentation is given for borehole waveguides, aiming at estimating the efficiency of an in-hole source for various surrounding media. The time-harmonic and transient solutions are obtained in terms of inverse Fourier integrals. The tube, body, and leaky waves propagating from the source to infinity are derived from those path integrals as an asymptotic contribution of the residues, stationary points, and approaching complex poles and stationary points. On this basis, the wave energy radiation from the source, which is dependent on the geometry and material properties of a cased fluid-filled borehole and formation, is numerically studied. The analysis is focused on the peculiarities of the source power distribution among the generated waves as well as between the radial and axial directions of wave propagation. A qualitative difference in source energy radiation modes for hard and soft environments and the role of leaky waves in the power transport into a soft formation are discussed. The emergence of additional tube waves due to a fluid saturation of porous interlayers is revealed.

1. Introduction

The objective of the study is wave generation and propagation in a circular coaxial multilayered cylindrical waveguide embedded in surrounding elastic or porous-elastic medium. The need for such studies appears in various technical applications ranging from nanotubes, optical fibers and fiber-reinforced composites to industrial pipelines and boreholes.

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The simulation of wave processes in all such structures is reduced to the same mathematical problems implying a common approach to their solution. For clarity and concreteness of presentation, the results of the present paper are given in terms and units of borehole seismic acoustics.

A downhole seismo-acoustic source generates in a borehole structure both body and tube waves radiated into the reservoir and propagating along the well. When it is used to increase the productivity of the reservoir, the source efficiency is determined, first, by the amplitude (power) of body waves excited in the bed, and, second, by the source energy partition between the useful body waves and useless tube ones. In this context, guided leaky waves that re-radiate wave energy into the formation in the course of their propagation along the tube are also of interest [1]. The opposite problem of minimizing the loss of tube wave energy by suppressing its leakage into the ground occurs in the long-range guided wave testing of buried pipelines [2]. In both cases, the evaluation of the wave energy transfer requires the development of efficient mathematical and computer models.

Theoretical investigation of wave propagation in cylindrical fluid-filled borehole structures goes back to the pioneering works by Biot [3] and Somers [4]. Traditionally, it relies on the modal analysis technique and ray methods of the geometrical diffraction theory. Such approaches operate with physically evident wave expressions in which the tube wave velocities are roots of transcendent characteristic equations while spatial eigenforms are eigensolutions associated with those roots. Starting from the wave analysis in empty cylindrical boreholes [5,6], the modal analysis has provided a comprehensive information about the characteristic features of various tube waves that may exist in fluid-filled boreholes (generalized Stoneley, pseudo-Rayleigh, etc.) as well as analytical studies of synthetic and experimental acoustic seismograms [7–11]. Unfortunately, the modes of modal expansions, being eigensolutions of boundary value problems with homogeneous boundary conditions, can be obtained only up to constant factors. Thus, they cannot be used for the evaluation of source energy partition among the excited modes until the modal expansion coefficients have been determined, e.g., via docking of such expansions with near-field solutions obtained by the finite element method (FEM). Similarly, ray expansions for body waves radiated into the formation have to be also stitched with the direct source field in a near-well zone.

The advantage of numerical methods is that they can simulate wave phenomena in complex borehole structures [12–15], including boreholes in poroelastic formations [16,17], or form the basis for hybrid FEM/analytic schemes for cylindrical waveguides of arbitrary structure and cross-section [18,19]. However, with models featured by relatively high frequencies, short wavelengths, and long wave propagation distances, considerably small time marching step and dense discretization can make the target problem too computationally expensive.

To develop low-cost models, in the present work we use the approach based on the Fourier transformation with respect to the guide's axis. It allows obtaining exact integral representations for the wave field generated by an intratube source in an arbitrarily laminated coaxial circular cylindrical channel and external formation in terms of inverse Fourier path integrals. The tube, body, and leaky waves generated by the source are extracted from those integrals using the residue technique and asymptotic methods. An important feature of those asymptotic representations is that, unlike the modal and ray expansions, their coefficients are already uniquely determined through the source-related factors in the integrands.

The use of integral representations for the field generated in a cylindrical structure by a given source goes back to the last century [20–26] and continues to be relevant until now [27] due to the advantages mentioned above. The integral approach has been used even with such complex structures as a borehole in an anisotropic formation [28] or with internally rough walls [29], multilayered [30] and porous fluid-saturated materials [31–34].

In this way, some analytical evaluations of a downhole source efficiency have been conducted [35]. However, in general, the source energy efficiency and the source energy partition among the excited waves are poorly studied yet.

The present paper is focused on the investigation of wave energy transfer from the source and energy partition among the excited waves in order to evaluate the efficiency of a downhole source located deep enough to neglect the waves reflected from the ground surface. Here we intensively use the analytical technique developed for the evaluation of wave energy fluxes in elastic multilayered structures [36,37]. One more goal is to discuss the leaky wave phenomenon, which is not well-studied, especially in connection with the energy transfer. Numerical examples, obtained in the context of the mathematical and computer models developed, illustrate the appearance of additional tube and leaky waves due to the lamination and fluid-saturated porosity and quantitatively different patterns of energy fluxes for boreholes in soft and hard formations.

2. Mathematical model

2.1. Problem statement

A pipe or well waveguide in a soil consists of three main components: an intratube cylinder D_1: $0 \le r \le b_1$ filled by fluid, a layered tube wall D_2: $b_1 \le r \le b_2$ (casing), and surrounding medium D_3: $b_2 \le r < \infty$ (formation) that may be elastic or porous elastic; $0 \le \theta \le 2\pi$, $-\infty < z < \infty$ (Fig. 1(a)). Here (r, z, θ) are cylindrical coordinates, the axis Oz is the central axis of the tube: $x = r\cos\theta, y = r\sin\theta, z = z$; $r = \sqrt{x^2 + y^2}$.

The wave field generated by the source is described by the displacement vector $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$. With a point monopole source located on the axis Oz, the field is independent of θ (axisymmetric), so the consideration is limited by the domain $D = D_1 \cup D_2 \cup D_3$: $r \ge 0$, $|z| < \infty$ in the cross-section plane (r, z) (Fig. 1(b)), where spherical coordinates (R, ψ, θ): $r = R\cos\psi$, $z = R\sin\psi$, $R = \sqrt{r^2 + z^2}$, $|\psi| < \pi/2$ will be also used. A relatively thin subdomain D_0: