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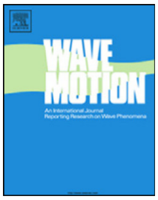

Sound absorption and reflection from a resonant metasurface: Homogenisation model with experimental validation

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HIGHLIGHTS

- Effective surface admittance derived by two-scale asymptotic homogenisation.
- Analytical solution used to design metasurface for sound control at resonance.
- Experimental procedure for metasurface characterisation is developed.
- Total sound absorption at resonance observed in the impedance tube below 400 Hz.
- Near-total sound absorption shown in anechoic chamber with finite size metasurface.

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ABSTRACT

Efficient manipulation of sound waves by some resonant acoustic metasurface designs has recently been reported in the literature. This paper presents a general theoretical framework for the description of sound wave interaction with the resonant metasurface that is independent of the nature of resonators and the excitation. The equations governing the behaviour of the metasurface are upscaled from the rigorous description of its unit cell using the two scale asymptotic homogenisation. The procedure relies on the existence of the boundary layer confined in the vicinity of the resonators operating in the deep subwavelength regime. The model is capable of describing sound interaction with the array of resonators positioned above or upon the substrate, so that the out of plane direction becomes an additional degree of freedom in the design. It is shown that at the leading order, the behaviour of the resonant surface is described in terms of the effective admittance, whose unconventional properties makes it possible to achieve the total sound absorption at multiple frequencies, broadband absorption, the phase reversal of the reflected wave at resonance and the control of the enclosure modes. The theory is validated by experiments performed in the impedance tube and in the anechoic environment using a surface array of spherical Helmholtz resonators with the extended inner neck. Experimental results confirm the effectiveness and robustness of the resonant surface for control of sound waves.

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1. Introduction

This article is devoted to the theoretical and experimental study of a resonant surface, that is a two-dimensional array of resonators arranged at a surface in a regular lattice. Both resonators and the spacing between them are small compared to the wavelength at the resonance frequency (condition of scale separation). Resonant surfaces are of interest in many domains of physics. They can be used, for instance, to control wavefields through tunable boundary conditions [1–3], or to achieve the perfect absorption of an incident wave [4,5]. In acoustics, examples of resonant surfaces include perforated panels with air cavity behind [6], quarter wavelength tubes folded in a bulky shape [7] or loaded membranes arranged above gas-filled cavity [5]. While the frequency selective absorption is often the targeted function of such surface arrays, other applications are emerging. One of them is to use arrays with graded properties to alter the phase of the reflection coefficient and realise wavefront manipulation [7], somewhat similarly to room diffusers [8]. While resonant surfaces are usually effective in a narrow frequency range close to the resonance, the use of several mistuned resonators per period can broaden the working frequency range [9], but it complicates the design as the period has to remain small compared to the wavelength.

Although several structures have been shown to achieve some remarkable effects in response to some particular incident wavefields (usually plane waves at normal incidence), it seems that a theoretical framework suitable for the description of surface effects in a general case is still lacking. This subject is addressed in this work. Here, a general analytical model based on the method of two-scale asymptotic homogenisation [10–12] is presented which can be applied to any acoustic resonant surface arranged on a rigid backing. No particular assumptions are made about the nature of the resonators or the incident field, except for the condition of scale separation. While the theory of two-scale asymptotic homogenisation is usually applied to find the effective constitutive laws of periodic bulk media, here it is used to derive the effective boundary condition satisfied by the propagating field at the surface array [13–20] by means of a boundary layer analysis. The model provides a systematic analysis of total sound absorption at single or multiple frequencies, broadband absorption, a phase reversal of reflected field at resonance, and control of enclosure mode. In particular, the model addresses the case when the resonators are positioned *above* the rigid backing, which makes the out-of-plane direction an additional degree of freedom in the design. The theory is validated experimentally using surface arrays of Helmholtz resonators with an extended inner duct designed in [21,22]. This design allows for a compact stacking of the resonators at the surface. The experiments are performed in both impedance tube and an anechoic chamber, with the former providing a basic technique for unit cell characterisation and the latter used to characterise absorption by finite-size surfaces.

The article is organised as follows. The two-scale asymptotic homogenisation method is presented in Section 2 and is used for the derivation of an equivalent surface admittance. The unconventional effects accompanying sound reflection from the structured surface are described in Section 3. The approach to the design of the resonant surface is described in Section 4, and the experimental results are presented in Sections 5 and 6.

2. Resonant surface admittance

In this section, the propagation of air-borne acoustic waves under ambient conditions is studied in the presence of the resonant surface and in the linear harmonic regime. At equilibrium, the density of air is $\rho_e = 1.213$ kg/m^3, the atmospheric pressure $P_e = 1.013 \times 10^5$ Pa, the adiabatic constant $\gamma = 1.4$, the thermal conductivity $\kappa = 0.026$ W/(Km), the heat capacity at constant pressure $c_p = 1.005 \times 10^3$ J/(kg K), the viscosity $\mu = 1.85 \times 10^{-5}$ Pa s, and the sound speed $c = \sqrt{\gamma P_e/\rho_e} \approx 342$ m/s. The study is performed at frequencies $\omega/2\pi$ (time convention $e^{-\mathrm{i}\omega t}$) close to the natural frequency of the resonators arranged on the surface. Following the model of Boutin & Roussillon developed for applications in elastodynamics [16], the acoustic behaviour of the resonant surface is described in terms of an effective surface condition derived by means of the two-scale asymptotic homogenisation [10–12]. The approach relies on the scale separation between the long wavelength at resonance and the characteristic lengths of the surface micro-structure, which leads to the existence of a boundary layer confined in the vicinity of the surface array [13–20].

2.1. Scale separation and boundary layer

The array under study is the 2-D periodic repetition of the Representative Elementary Volume (REV) Ω at a plane surface $\mathcal{S}$, with inward normal vector $\mathbf{n}$ directed at air. The REV Ω includes N linear acoustic resonators and the rigid backing, see Fig. 1. The nature of the resonators is not specified at this stage. They can be positioned upon or above the rigid backing as well as fully or partially embedded in it. In order to describe this array of *three-dimensional* REVs by an equivalent *surface* condition, the homogenisation process must be performed along with a reduction of one space dimension. To do so, the following condition of scale separation is required: the characteristic size ℓ of the REV (microscopic scale) has to be much smaller than the reduced wavelength $L = \lambda/(2\pi)$ (macroscopic scale) of the propagating field in air. This condition is quantified by the scale parameter $\epsilon = \ell/L \ll 1$ which defines the applicability of the model.

The resonators in the REVs experience resonance within the frequency range of scale-separation. To satisfy this condition, a specific design of resonators is required, that leads to a regime of *co-dynamics* [21] between the microscopic and macroscopic scales. This regime has been described as early as 1985 by Auriault & Bonnet in elastodynamics [23] (see Ref. [24] for an English version). Since the 2000s, the specific features of this regime have been extensively used to engineer