Wave Motion 72 (2017) 244–259

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Coherent wave propagation in viscoelastic media with mode conversions and pair-correlated scatterers

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HIGHLIGHTS

- The effective wavenumber is expanded up to order 3 in concentration (c).
- The effect of correlation between scatterers is studied.
- It is of order 3 in concentration but of order 2 in scattering.
- It can modify greatly the wave velocity around a sub wavelength resonance.
- It increases the attenuation by a small constant amount at higher frequency.

ARTICLE INFO

Article history:
Received 14 June 2016
Received in revised form 14 December 2016
Accepted 12 March 2017
Available online 22 March 2017

Keywords:
Scattering
Correlation
Coherent wave
Ultrasound

ABSTRACT

The influence of correlation between scatterers on coherent waves propagation is studied in the case of a viscoelastic medium hosting a random configuration of either spherical or cylindrical scatterers. A distinction is made between the hole correction and the additional disturbances to the pair correlation function beyond the excluded volume *via* a radial and concentration dependent Ursell function. The effect of the Ursell function on the effective wavenumber is shown to be of order 3 in concentration and order 2 in scattering, and the corresponding formulas generalize those of Caleap et al. (2012) for an ideal fluid host medium. The whole order 3 in concentration is calculated; its other part is of order 3 in scattering. Both parts of the order 3 in concentration are the sum of two terms, one related to mode conversions, the other not. The numerical study is performed mostly for aluminum spheres in epoxy, which is a rather illustrative situation of the different phenomena that participate to the coherent propagation. The Ursell function effect is enhanced at low frequency, while counteracted partly at higher frequency, by the other term of order 3 in concentration. The most visible effects of both terms are on the attenuation. The Ursell term related to mode conversions is larger than the one with no mode conversions included in the low frequency regime.

1. Introduction

Multiple scattering of waves in random media is of practical interest in many fields, ranging from ultrasonic monitoring of particulate mixtures [1] to the detection of flaws in industrial [2], biological [3,4], or other natural media [5]. Metamaterials

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http://dx.doi.org/10.1016/j.wavemoti.2017.03.002
0165-2125/

are often built by the periodic arrangement of scatterers in a host matrix [6–8], but it is also possible to get interesting properties from random ones [9]. In such cases, the concentration of scatterers is rather large (approximately 20%) and it becomes important to have models well adapted to such concentrations.

While the well-known effective-field theories [10,11] provide pretty good approximations of the wavenumber of the coherent wave in sparse concentrations of scatterers in an ideal host fluid, they fail at larger concentrations [12], and people usually either turn to other theories, such as the Coherent Potential Approximation [13], or introduce a pair distribution function [14,15] that takes into account the average microstructure of the random medium [16]. Self-consistent theories have been developed also for elastic media [17] (see references therein), but their main drawback is that they lead to a characteristic equation that is to be solved numerically to obtain the coherent wavenumbers. This is the reason why we rather consider here the effect of a pair distribution function on the formulas for the wavenumber obtained in the frame of an effective field-theory. Caleap's study [15] was conducted for spheres in an ideal fluid, and the aim of this paper is to conduct a similar one for host media in which more than one type of wave may propagate: elastic, porous, thermo-visco-elastic media.…

Section 2 is thus devoted to the description of the multiple scattering medium, consisting in a random distribution of either identical spheres (3D case) or identical infinitely long cylinders (2D case) in an isotropic host medium in which P different types of waves may propagate. The pair distribution function is the sum of a hole correction term and a radial concentration dependent Ursell function. The dispersion equation of the P coherent waves is established in Section 3, in which the 3D case only is detailed, the corresponding useful equations of the 2D case being given in Appendix A. The analytic expansion of the effective wavenumbers is performed in Section 4 up to power 3 in concentration, and it is shown in Section 5 that the term due to the Ursell function is one order less in scattering than in concentration. Finally, numerical simulations are performed in Section 6 in order to compare the relative importance, on the coherent wave properties, of the hole correction and the Ursell function.

2. Statistics of the medium

We consider $N = n_0 V$ identical spheres of radius a randomly distributed in a given region of volume V with a concentration c equal to $(4/3)\, n_0 \pi a^3$. The probability density of finding one sphere centered at $\mathbf{r_2}$ if one is known to be at $\mathbf{r_1}$ is

$$p\left(\mathbf{r_2}|\mathbf{r_1}\right) = \frac{1}{N-1} n\left(\mathbf{r_2}|\mathbf{r_1}\right). \tag{1}$$

The simplest choice for the conditional number density $n\left(\mathbf{r_2}|\mathbf{r_1}\right)$,

$$n\left(\mathbf{r_2}|\mathbf{r_1}\right) = \begin{cases} n_0 & \text{for } \|\mathbf{r_2} - \mathbf{r_1}\| > b \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

is known as the "Hole Correction". Usually, b is assumed to satisfy $b > 2a$ so that the spheres are not allowed to overlap. However, it is known that the hole correction, whilst physically reasonable for sparse concentration, becomes a poor approximation with the increase of concentration. More generally we can use [16]

$$n\left(\mathbf{r_2}|\mathbf{r_1}\right) = \begin{cases} n_0 \quad \left(1 + U\left(r, n_0\right)\right) & \text{for } \|\mathbf{r_2} - \mathbf{r_1}\| > b \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The U function, in addition to decaying rapidly to zero as $r = \|\mathbf{r_2} - \mathbf{r_1}\|$ tends to infinity, obeys

$$\lim_{n_0 \to 0} U\left(r, n_0\right) = 0. \tag{4}$$

The pair distribution function $1 + U\left(r, n_0\right)$ of radially symmetric scatterers such as those considered here is also known as the "radial distribution function", and $U\left(r, n_0\right)$ is called the (two-particle) Ursell function [18,19]. There are many approximations of the radial distribution function for natural random distributions of solid spherical particles at equilibrium in an isotropic medium, depending on the number and type (thermo dynamical, or mechanical) of interaction forces between particles and surrounding medium. The most well known is the Percus–Yevick approximation, which is a solution of the exact Percus–Yevick equation for the description of a classical fluid, and which is shown in Ref. [15] to describe the average over a large number of realizations of artificial random distributions as well. The Ursell function is obviously negligible for sparse concentration, as shown with Eq. (4).

Studying the effect of the pair distribution function term on the effective wavenumber expansion into powers of the concentration has been done in Ref. [15] for an ideal fluid host medium. The next sections use Refs. [15,20,21] in order to extend this study to host media in which mode conversions may occur during scattering. The different steps of the calculation are detailed for the 3D case only, but the final useful formulas are given in both cases in the Appendices.