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A two-way model for nonlinear acoustic waves in a non-uniform lattice of Helmholtz resonators

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HIGHLIGHTS

- A tube connected with an array of Helmholtz resonators constitutes a dispersive medium, leading to acoustic solitary waves.
- A model is proposed to describe the phenomena at hand.
- The contribution is to model the individual features of each resonators and the two-way propagation.
- Doing so allows to investigate the interaction of solitary waves with defects, and their propagation in random media.

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ABSTRACT

Propagation of high amplitude acoustic pulses is studied in a 1D waveguide connected to a lattice of Helmholtz resonators. An homogenized model has been proposed by Sugimoto (1992), taking into account both the nonlinear wave propagation and various mechanisms of dissipation. This model is extended here to take into account two important features: resonators of different strengths and back-scattering effects. An energy balance is obtained, and a numerical method is developed. A closer agreement is reached between numerical and experimental results. Numerical experiments are also proposed to highlight the effect of defects and of disorder.

1. Introduction

We are concerned here with the dynamics of nonlinear waves in lattices. This subject has stimulated researches in a wide range of areas, including the theory of solitons and the dynamics of discrete networks. Numerous studies have been led in electromagnetism and optics [1], and many physical phenomena have been highlighted, such as discrete breathers [2–4], chaotic phenomena [5,6], dynamical multistability [7,8] and solitons or solitary waves [9,10]. We focus more specifically on solitary waves, which have been first observed and studied for surface waves in shallow water [11]. The main feature of these waves is that they can propagate without change of shape and with a velocity depending of their amplitude [12]. They have been studied in many physical systems, as in fluid dynamics, optics or plasma physics.

For elastic waves, numerous works have highlighted the existence of solitary waves in microstructured solids [13], periodic structures such as lattices or crystals [14–16], elastic layers [17–19], layered structures coated by film of soft material [20], periodic chains of elastics beads [21–23] and uniform or inhomogeneous rods [24–26].

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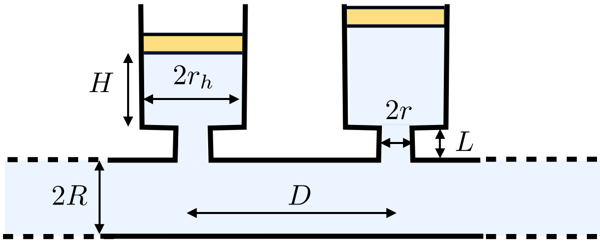

Fig. 1. Sketch of the guide connected with Helmholtz resonators.

On the contrary, only a few works have dealt with acoustic solitary waves. Such a lack is mainly explained by the fact that the intrinsic dispersion of acoustic equations is too low to compete with the nonlinear effects, preventing from the occurrence of solitons. To observe these waves, geometrical dispersion must be introduced. It has been the object of the works of Sugimoto and his co-authors [27–30], where the propagation of nonlinear waves was considered in a tube connected to an array of Helmholtz resonators. A model incorporating both the nonlinear wave propagation in the tube and the nonlinear oscillations in the resonators has been proposed. Theoretical and experimental investigations have shown the existence of acoustic solitary waves [27]. We have developed a numerical modeling, and we have successfully compared simulations with experimental data [31,32].

One fundamental assumption underlying Sugimoto's model is that all the resonators are the same, which allows to use a homogenization process. The drawback of this approach is that the reflection of an incident wave by a defect (for instance, a resonator different from the others) cannot be considered. Similarly, wave propagation across resonators with variable features cannot be investigated. However, the case of variable resonators is important when studying the influence of manufacturing defects or the influence of aging of the guiding device on the wave propagation.

The aim of this paper is to remedy these limitations, by building a model predicting two-way propagation across variable resonators. For this purpose, we introduce a discrete description of the resonators. A consequence is that the requirement of a long wavelength is no longer necessary. Second, we allow the reflection of waves. Finally, we reformulate the viscothermal losses, which leads to a formulation suitable for an energy balance. The new model writes as a single system coupling three unknowns, the velocity of the right-going wave u^+, the velocity of the left-going wave u^- and the total excess pressure in the resonators p, induced by both waves:

$$\begin{cases} \dfrac{\partial u^+}{\partial t} + \dfrac{\partial}{\partial x}\left(a_0 u^+ + b\dfrac{(u^+)^2}{2}\right) + \dfrac{c}{a_0}\dfrac{\partial^{1/2}}{\partial t^{1/2}}u^+ - d\dfrac{\partial^2 u^+}{\partial x^2} = -e(1-2mp)\dfrac{\partial p}{\partial t}, \\ \dfrac{\partial u^-}{\partial t} + \dfrac{\partial}{\partial x}\left(-a_0 u^- + b\dfrac{(u^-)^2}{2}\right) + \dfrac{c}{a_0}\dfrac{\partial^{1/2}}{\partial t^{1/2}}u^- - d\dfrac{\partial^2 u^-}{\partial x^2} = +e(1-2mp)\dfrac{\partial p}{\partial t}, \\ \dfrac{\partial^2 p}{\partial t^2} + f\dfrac{\partial^{3/2}}{\partial t^{3/2}}p + gp - m\dfrac{\partial^2 (p^2)}{\partial t^2} + n\left|\dfrac{\partial p}{\partial t}\right|\dfrac{\partial p}{\partial t} = h(u^+ - u^-). \end{cases} \tag{1}$$

The precise meaning of $u^\pm$, p and the coefficients in (1) will be detailed along Section 2. Some of the coefficients incorporate the individual features of the resonators and vary with x.

The paper is organized as follows. In Section 2, the general equations in the tube and in the resonators are given, and the new model (1) is derived. In Section 3, a first-order formulation is followed: it allows to determine an energy balance and also to build a numerical scheme. In Section 4, comparisons with experimental data show that a closer agreement is obtained than with the original Sugimoto's model. Numerical simulations are performed to investigate the properties of the acoustic solitary waves, in particular the robustness to defects and to random disorder. Conclusions and prospects are drawn in Section 5. Technical results are given in the Appendices A–C.

2. The new model

In this part, we derive Eq. (1). The configuration under study is made up of an air-filled tube connected to uniformly distributed cylindrical Helmholtz resonators (see Fig. 1). The geometrical parameters are the radius of the guide R, the axial spacing between successive resonators D, the radius of the neck r, the length of the neck L, the radius of the cavity r_h and the height of the cavity H, which can vary from one resonator to the other. The cross-sectional area of the guide is $A = \pi R^2$ and that of the neck is $B = \pi r^2$, the volume of each resonator is $V = \pi r_h^2 H$.

2.1. Equations in the tube

2.1.1. General equations

Here we recall briefly the general equations governing the nonlinear acoustic waves in a tube [28]. The physical parameters are the ratio of specific heats at constant pressure and volume γ; the pressure at equilibrium p_0; the density