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Perforated grating stacks in thin elastic plates

Michael H. Meylan [a,*], Michael J.A. Smith [b]

[a] School of Mathematical and Physical Sciences, The University of Newcastle, New South Wales 2308, Australia
[b] CUDOS ARC Centre of Excellence, School of Physics, University of Sydney, Sydney, New South Wales 2006, Australia

HIGHLIGHTS

- A boundary-element method for grating stacks in thin elastic plates is given.
- The numerical solution is validated against circular cavities solved by the multipole method.
- Results are presented for various cavity geometries.

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ABSTRACT

The reflection and transmission spectrum of an individual grating, and repeated stacks of gratings, is computed using boundary integral methods for arrays of arbitrarily shaped cavities, with free-edge boundary conditions, that are periodically repeated in a thin elastic plate. The solution is found using a specially developed boundary element method coupled with an array Green's function. The computational code is tested against the solution obtained using multipoles which applies only to circular geometries but is shown to give very good agreement. The code is also validated using energy balance equations which includes the evanescent modes. A number of cavity shapes are investigated and results are compared against circular cavities with equivalent volumes.

1. Introduction

Diffraction gratings are periodic structures that provide active control over the propagation of waves through media. They are extensively used in a wide range of scientific fields, with particular emphasis in the fields of optics, acoustics, chemistry and in the life sciences [1]. Through a careful choice of grating design, the controlled manipulation of light, sound, and vibrational waves can be achieved to build a wide of practical devices and to steer waves in specified directions. Despite the extensive attention paid to photonic and phononic structures, wave propagation in different systems, such as thin elastic plates, has received comparatively little research attention. This is widely regarded as being due to the extensive range of possibilities already open in photonics and phononics, but also because the governing equation for thin plates is the biharmonic operator, which is a fourth-order partial differential equation, in contrast to the Helmholtz equation, which is a second-order equation that governs wave dynamics in a wide range of periodic systems.

The study of wave propagation through thin elastic plates is commonly referred to as 'platonics' [2], and in this work we will consider plane wave propagation through gratings comprising a one-dimensional array of inclusions of arbitrary geometry. At the edges of these inclusions, free-edge boundary conditions are imposed, in an extension of previous work by some of the authors [3] on two-dimensional platonic crystals and platonic gratings with clamped-edge boundary conditions.

* Corresponding author.
E-mail address: mikemeylan@icloud.com (M.H. Meylan).

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Existing work on platonic structures has been primarily restricted to clamped and free-edge circular inclusions [4–6] or to zero-radius clamped inclusions (pins) [7–12], where semi-analytic solutions are available using multipole methods. That said, other geometries such as squares [13,14] and circular inclusions bisected with Euler–Bernoulli beams [15] have also been considered. The fundamental issue with free-edge boundary conditions is that they are intractable, even for circular inclusions, which has given rise to incorrect expressions for free-edge platonic crystals [5]. This error was found in the development of the present work, which lead to the corrected form for two-dimensional arrays of circular inclusions in [16], and in the Appendix to the present work we include the correct multipole solution for a grating of free-edge circular inclusions to avoid any possible ambiguity.

The solution method we present is a generalized boundary-integral approach [3,17]. In this setting, the periodicity of the system is embedded in quasi-periodic Green's functions which in their canonical form, are slowly converging. However a wide range of methods have been developed to accelerate their convergence [18] and these are employed in the present work. After imposing the free-edge boundary conditions in the boundary integral system, we evaluate the unknown boundary displacement and normal derivative as Fourier series and impose a solvability condition. This admits a matrix system for the unknown coefficients and gives the necessary boundary data to evaluate the reflection and transmission coefficients for the grating problem [19,3]. Incidentally, the fundamental difference between a grating and a crystal is that the domain for the grating problem is geometrically non-compact, and so there is no discrete spectrum, even though a crystal can be regarded as an infinite stack of gratings.

We emphasize that the numerical procedure presented here is intensive, and the results presented primarily serve to illustrate the method. The boundary integral method shown here is able to give a converged solution for the platonic grating problem, and we note that at present, commercially available finite element solvers have no direct framework for a solution. To obtain a finite element method solution to a one-dimensional grating problem, a finite domain must be considered, and imposing the appropriate radiation condition (i.e., imposing perfectly matched layers for the biharmonic operator) is expected to be a non-trivial task. We have developed a complete solution method for arbitrary geometries where this difficulty is entirely avoided as the Green's functions satisfy the necessary radiation conditions.

In addition to considering wave scattering by platonic gratings, we also consider stacks of gratings [20,21], and present a well-known recurrence relation procedure to determine the reflection and transmission matrices for multiple layers. For a selection of arbitrary geometry configurations (where the inclusion areas are approximately equivalent) we demonstrate that the principal orders of reflection are periodic with increasing rectangular spacing. For a single grating comprising arbitrarily shaped inclusions with two- and four-fold symmetry, the choice of inclusion geometry has a minimal effect on the transmission and reflection spectrum. However, for more complicated geometries such as Helmholtz resonators, the reflection spectra exhibits unique characteristics.

The outline of this paper is as follows. In Section 2 we present the problem formulation for wave incidence on an arbitrary grating for all boundary condition classes. In Section 3 we consider the solution for an arbitrary inclusion inside a grating cell. In Section 4 we decompose the boundary data into Helmholtz and modified Helmholtz components for use in Section 5 where we compute the reflection and transmission matrices for a single grating. In Section 6 we outline the recurrence relation procedure for multiple grating stacks. In Section 7 we present the energy balance equation for platonic gratings. This is followed in Section 8 by a selection of numerical results and concluding remarks in Section 9. Appendix presents the multipole solution for a grating of circular, free-edge inclusions.

2. Problem formulation

The governing equation for the out-of-plane displacement of a thin, elastic plate (Kirchhoff–Love plate) is the biharmonic equation

$$(\Delta^2 - k^4)w = (\Delta + k^2)(\Delta - k^2)w = 0, \tag{1}$$

where $k^2 = \omega\sqrt{\rho h/D}$, ω is the angular frequency, ρ is the mass density, h is the thickness, D is the flexural rigidity of the plate, and we have assumed a time dependence of $\exp(-\mathrm{i}\omega t)$. In this paper we consider plane wave incidence upon an array of cavities of identical shape (but arbitrary geometry) assuming free-edge boundary conditions are imposed on the edges of each hole. Following the procedure in Smith et al. [17,3], this system of equation can be decomposed into two natural field components, and we provide a brief outline for the purposes of completeness here.

Due to the linearity of the biharmonic equation the total displacement can be decomposed into incident and scattered fields, which may also be decomposed into their respective Helmholtz and modified Helmholtz components as

$$w = w_{\mathrm{I}}^{\mathrm{H}} + w_{\mathrm{S}}^{\mathrm{H}} + w_{\mathrm{I}}^{\mathrm{M}} + w_{\mathrm{S}}^{\mathrm{M}}. \tag{2}$$

Using Green's second identity with this decomposition, a boundary integral system describing a single grating is obtained [3,17] and takes the form

$$\tfrac{1}{2} w^{\mathrm{M}}(\mathbf{x}) = w_{\mathrm{I}}^{\mathrm{M}}(\mathbf{x}) + \sum_{m=-\infty}^{\infty} \int_{\partial\Omega_m} \left\{ \partial_{n'} G^{\mathrm{M}}(\mathbf{x},\mathbf{x}') w^{\mathrm{M}}(\mathbf{x}') - \partial_{n'} w^{\mathrm{M}}(\mathbf{x}') G^{\mathrm{M}}(\mathbf{x},\mathbf{x}') \right\} \mathrm{d}S', \tag{3a}$$

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