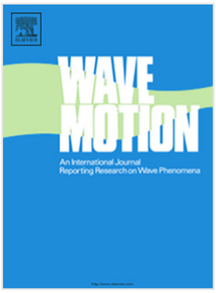
Accepted Manuscript



Wave–wave interactions in a periodic chain with quadratic nonlinearity

Smruti R. Panigrahi, Brian F. Feeny, Alejandro R. Diaz

PII: S0165-2125(16)30155-X
DOI: http://dx.doi.org/10.1016/j.wavemoti.2016.11.008
Reference: WAMOT 2126

To appear in: *Wave Motion*

Received date: 1 May 2016
Revised date: 8 November 2016
Accepted date: 17 November 2016

Please cite this article as: S.R. Panigrahi, B.F. Feeny, A.R. Diaz, Wave–wave interactions in a periodic chain with quadratic nonlinearity, *Wave Motion* (2016), http://dx.doi.org/10.1016/j.wavemoti.2016.11.008

Wave-Wave Interactions in a Periodic Chain with Quadratic Nonlinearity

Smruti R. Panigrahi*a, Brian F. Feenya, and Alejandro R. Diaza
a428 S. Shaw Lane, Michigan State University, East Lansing, MI, 48824, USA

Abstract

Two waves are studied using perturbation analysis for their interactions in an one-dimensional periodic structure with quadratic nonlinearity. A first-order multiple-scales analysis along with numerical simulations on the full chain are used to understand the interaction of two waves when one is the sub- or super-harmonic of the other. The strength of quadratic nonlinearity affects the rate at which the energy is exchanged between the two waves. Depending on parameters and energy states, the interactions between the waves are periodic or whirling and result in quasi-periodic combined propagating waves with either phase drifts or weakly phase-locking properties. The analysis suggests the possibility of the existence of emergent wave harmonics. Due to quadratic nonlinearity, a very small amplitude subharmonic or superharmonic wave mode can drift in its phase, and then burst out with a larger amplitude as it circumnavigates a separatrix. Depending on the parameters and wave numbers, the amplitude of this emergent wave burst can have varying significance.

Research highlights

- Interactions between two waves traveling in a one-dimensional periodic chain with quadratic nonlinearity have been investigated.
- When one wave is the super- or sub-harmonic of the other, the phase space dynamics shows energy exchange between the waves.
- The quadratic nonlinearity affects the rate at which the energy is exchanged.
- The interaction of the waves show periodic or whirling behavior depending on the parameters and initial energy state.
- The quadratic nonlinearity induces phase-drift and weakly phase-locking properties.

Keywords: Traveling waves, periodic structures, quadratic nonlinearity, perturbation, method of multiple scales, dispersion, metamaterials, acoustic filters, subharmonic, superharmonic, emergent waves, phase drift, phase lock.

I. INTRODUCTION

Due to the presence of bandgaps in periodic media, many researchers have been focusing on wave propagation in nonlinear periodic structures and its application to the design of novel metamaterials [1]–[8]. Structures exhibiting bandgaps prevent the propagation of waves at certain frequencies. These structures may be phononic (sonic) or photonic, depending on their band-gap frequency range. Phononic or sonic band-gap structures can be used as sensing devices based on resonators, acoustic logic ports and wave guides, and frequency filters based on surface acoustic waves, while photonic band-gap structures have applications in optics and microwaves. Synthesis of phononic materials with desired band-gap and wave-guiding characteristics has been achieved through the application of topology and material optimization procedures [9]–[11]. The application of periodic plane grid structures as phononic materials and its design optimization process have been presented in [12], where a limited number of continuously varying parameters define the geometry of a predefined cellular topology that deals with periodic structures of infinite size as well as demonstrate the validity of the results to finite systems.

Wave-wave interaction and its evolution in a continuum have been investigated [3], [13], [14] including theoretical study on the interaction of harmonic elastic plane waves in a cubically nonlinear material using the method of multiple scales [3]. The frequency vs. wave number dispersion relation of a one dimensional linear chain may be confined to the first Brillouin zone due to spatial periodicity [15]. Nonlinearities play an important and crucial role in the dynamics of this type of devices especially in small scale applications (e.g. Micro-Electro-Mechanical-Systems) [16], [17]. Hence understanding the influence of nonlinearities will allow possibilities of enhancing the material properties of these special group of materials.

Amplitude-dependent dispersion and bandgap behavior have been explored in several discrete periodic systems characterized by cubic nonlinearities by Narisetti et al. [6], where it was shown that the boundary of the dispersion curve may shift with amplitude in the presence of a single plane wave. Manktelow et al. [18] have recently extended the analysis in [6] to include the propagation of multiple harmonic plane waves that show the dispersion properties of discrete, periodic, cubically nonlinear systems. They presented a comparison between the multiple scales and Lindstedt-Poincare method for harmonic wave-wave

---

*corresponding author: Controls Research Engineer and Data Scientist in Research & Advanced Engineering at Ford Motor Company. email: smrutiranjan@gmail.com.