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Union values for games with coalition structure☆

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ABSTRACT

In a *cooperative transferable utility game* each decision-making agent is usually represented by one player. We model a situation where a decision-making agent can be represented by more than one player by a *game with coalition structure* where, besides the game, there is a partition of the player set into several *unions*. But, whereas usually the decision-making agents are the players in such a game, in this paper the decision-making agents are modeled as the unions in the coalition structure. Consequently, where usually a solution assigns payoffs to the individual players, we introduce the concept of *union value* being solutions that assign payoffs to the unions in a game with coalition structure. We introduce two such union values, both generalizing the Shapley value for TU-games. The first is the *union-Shapley value* and considers the unions in the most unified way: when a union enters a coalition then it enters with all its players. The second is the *player-Shapley value* which takes all players as units, and the payoff of a union is the sum of the payoffs over all its players. We provide axiomatic characterizations of these two union values differing only in a collusion neutrality axiom. After that we apply them to airport games and voting games.

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1. Introduction

A *cooperative game with transferable utility*, or simply a TU-game, consists of a finite set of players and for every subset (coalition) of players a worth representing the total payoff that the coalition can obtain by cooperating. A (single-valued) solution is a function that assigns to every game a payoff vector which components are the individual payoffs of the players. One of the most applied solutions for cooperative TU-games is the *Shapley value* [26].

In a TU-game a player only has to decide on whether to cooperate within a coalition or not. However, real life situations of cooperation are in most cases more than just a question of participation. For example, a decision-making agent might have to decide on how much money to invest in a certain project. In this case there is a range of options that the decision-making agent can actually choose from. Another example is a situation where a manager within a department has to decide on which employees to put on a certain project.

The idea behind these examples is that a decision-making agent might have multiple 'assets' that it may be able to call on in a situation of cooperation. Therefore, instead of a cooperation situation being modeled solely by a game on the decision-making agents, we argue that it might be modeled on the assets that the decision-making agent may be able to employ (be it money, subordinates or something else), but that benefit should still be assigned to the decision-making agent itself.

To mention an example, an important application of cooperative games in cost allocation is the airport game of Littlechild & Owen [19], where different airplanes that want to use the same landing strip must pay landing fees that cover the cost of building and maintaining the landing strip (see also Littlechild & Thompson [20]). In these airport games the players are usually the airplane movements, i.e. every player represents the landing of one airplane, and a solution assigns a payoff to every player in an airport game. However, the real decision-making units are the airline companies whose assets are the landings of any of their airplanes. This indeed focuses on the assets that a decision-making agent is able to employ. However, it seems to be silently implied that an airline company is nothing more than the sum of its separate airline movements, without considering whether just summing is really a desirable way of answering the question of how to divide cost among airline companies. In some sense the question of how to divide benefits from different assets over the decision-making agents is ignored within classic TU-games.

This problem was partly adressed by Hsiao & Raghavan [13], and later extended by van den Nouweland et al. [22], in their papers on multi-choice TU-games. In these games the decision-making agent is assigned a different (finite) number of activity levels. Participation is no longer a yes/no decision but agents can decide on the activity level with which to enter a coalition, choosing from a discrete finite number

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of levels ranging from no activity to full activity. It can be argued that these different levels can be seen as assets that a decision-making agent can use in cooperating. A high level of activity (high activity asset) being present for a certain decision-making agent also implies the presence of lower levels of activity (lower activity assets), and so these games are suitable to model situations (e.g. money investment) where there exists a natural order on the assets that a decision-making agent may call on. They do however exclude a class of situations like the one with the airplane landings mentioned above, where the choice made by an airline for one airplane landing does not necessarily imply that of another airplane landing.

In classic TU-games, a decision-making agent is represented by exactly one player in the game. In this paper we apply *games with coalition structure* introduced by Aumann & Drèze [3] to model situations where agents can be represented by more than one player. In a game with coalition structure, the player set is partitioned into a number of disjoint unions. The union expresses that players belong to a common group (this might for example be a family, a sports team, a firm, etc.).[3] A solution for such games assigns a payoff to every individual player. Examples of such solutions are, e.g. the Owen value [24], the coalitional τ-value [7], the two-step Shapley value [15] and the collective value [16]. In our interpretation, decision-making agents (that can be represented by possibly more than one player) are modeled as the unions in such a game with coalition structure. Therefore, in our approach, a solution assigns a payoff to every union (instead of to every player) in the game. To avoid confusion with the usual interpretation we will refer to such solutions that assign payoffs to the unions as *union values* for games with coalition structure. We introduce two such union values, both being generalizations of the Shapley value for TU-games. The first is the *union-Shapley value* and considers the unions in the most unified way. It simply takes a union with all its players as one unit, and when a union enters a coalition then it enters with all its players. The other solution is called the *player-Shapley value* and takes the players as units in the cooperative situation. Here, the payoff of a union is the sum of the payoffs over all its players. By considering these two solutions we consider the above mentioned question of how to divide benefit from different assets, that we feel is being ignored in classic TU-games.

Interestingly, both solutions mentioned above satisfy efficiency, the null union property and, moreover, satisfy (different) collusion neutrality[4] properties. This is surprising since van den Brink [5] showed that there is no solution for TU-games that satisfies efficiency, the null player property and collusion neutrality. So, considering union values for games with coalition structure where decision-making agents are represented by more than one player, does allow for collusion neutrality properties that are compatible with efficiency and the null player property. In particular, we provide axiomatizations of the two Shapley type solutions mentioned above that differ only in the collusion neutrality axiom that is used. First, the union-Shapley value satisfies *player collusion neutrality* stating that collusion of two players belonging to the same union does not change the payoff of this union. On the other hand, the player-Shapley value satisfies *union collusion neutrality* stating that after a collusion of two unions, the sum of their payoffs does not change.

After axiomatizing the union- and player-Shapley values we apply them to airport games and voting games. In particular, for airport games we distinguish between the costs that depend on the size of the airplanes that are using the landing strip, and costs that do not depend on this. We argue that for one type of costs the union-Shapley value is a suitable solution, while for the other type of cost the player-Shapley value is more suitable. For weighted voting games, the union-Shapley value yields the 'traditional' Shapley value (or Shapley-Shubik index, see Shapley and Shubik [27]), often used as a measure assigning voting power to the different parties in parliament. The player-Shapley value simply assigns to every party the number of members in parliament, and is often used to distribute the ministries among the parties that form the government.

The paper is organized as follows. Section 2 contains preliminaries. After presenting the model and the two solutions in Section 3, we provide axiomatic characterizations of these solutions in Section 4. In Section 5 we apply these solutions to sharing costs in airport games and voting games. Finally, there is an appendix containing proofs and showing logical independence of the axioms used to characterize the two solutions.

2. Preliminaries

A situation in which a finite set of players $N \subset \mathbb{N}$ can generate certain payoffs by cooperation can be described by a *cooperative game with transferable utility* (or simply a TU-game), being a pair (N, v) where v: $2^N \to \mathbb{R}$, with $2^N = \{S \mid S \subseteq N\}$, is a *characteristic function* on N satisfying $v(\emptyset) = 0$. For any coalition $S \subseteq N$, $v(S) \in \mathbb{R}$ is the *worth* of coalition S, i.e. the members of coalition S can obtain a total payoff of $v(S)$ by agreeing to cooperate. We denote the collection of all characteristic functions on player set N by $\mathcal{G}^N$. When there is no confusion about the player set we sometimes refer to a characteristic function as game v.

A *payoff vector* for game (N, v) is an $|N|$-dimensional vector $x \in \mathbb{R}^N$, where $\mathbb{R}^N$ is the euclidean $|N|$-dimensional space with coordinates indexed by elements of N, assigning a payoff $x_i \in \mathbb{R}$ to any player $i \in N$. A (single-valued) *solution* for TU-games is a function that assigns a payoff vector to every TU-game. One of the most famous solutions for TU-games is the *Shapley value* [26] given by

$$Sh_i(N, v) = \frac{1}{|N|!} \sum_{\pi \in \prod(N)} m_i^{\pi}(v) \quad \text{for all } i \in N,$$

where $\Pi(N)$ is the collection of all permutations π: $N \to N$ and $m_i^{\pi}(v) = v(\{j \in N | \pi(j) \leq \pi(i)\}) - v(\{j \in N | \pi(j) < \pi(i)\})$ is the marginal contribution of player i to the coalition of players that precede i in permutation π. So, the Shapley value assigns to every player its expected marginal contribution if all permutations are equally likely.

For each nonempty $T \subseteq N$, $T \neq \emptyset$, the *unanimity* game $u_T \in \mathcal{G}^N$ is given by $u_T(S) = 1$ if $T \subseteq S$, and $u_T(S) = 0$ otherwise. It is well-known that the unanimity games form a basis of the vector space $\mathcal{G}^N$. For every $v \in \mathcal{G}^N$ it holds that $v = \sum_{\substack{T \subseteq N \\ T \neq \emptyset}} \Delta_v(T) u_T$, where $\Delta_v(T) = \sum_{S \subseteq T} (-1)^{|T|-|S|} v(S)$ are the *Harsanyi dividends* (see Harsanyi [12]).

A *game with coalition structure* (see e.g. Aumann & Drèze [3] and Owen [24]) is a triple (N, v, P) where $N \subset \mathbb{N}$ is the (finite) set of players, v is a characteristic function on the set of players N, and $P = \{P_1, \ldots P_m\}$ is a partition of N which is called a *coalition structure*. The elements of the partition P are called *unions*. The idea behind the partition P is that every P_k, $k \in \{1, \ldots m\}$, is a union consisting of several players who are, in some sense, more related to each other than to players of other unions. Given $P = \{P_1, \ldots, P_m\}$ we denote $M = \{1, \ldots, m\}$. We denote by $\mathcal{P}^N$ the collection of all partitions of N, and by $\mathcal{GP}$ the collection of all games with coalition structure.

Usually, a solution for games with coalition structure assigns to every game with coalition structure (N, v, P) a payoff vector in $\mathbb{R}^N$ where the i-th component is the payoff for player $i \in N$. One of the most famous solutions is the *Owen value* which is obtained by taking the average over those marginal vectors of game v where players in one union enter consecutively, i.e. the Owen value is given by

$$Ow_i(N, v, P) = \frac{1}{|\prod^P(N)|!} \sum_{\pi \in \prod^P(N)} m_i^{\pi}(v) \quad \text{for all } i \in N,$$

where $\prod^P(N) = \{\pi \in \prod(N) | \pi(i) < \pi(j) < \pi(h)$ and $\{i, h\} \subseteq P_k \in P$ implies that $j \in P_k\}$ is the collection of all permutations where players

[3] Models where every union again can be partitioned into subunions (which can be refined further and so on), are considered in, e.g. Charnes & Littlechild [8], Winter [30] and Álvarez-Mozos & Tejada [2].

[4] We refer to the collusion neutrality type of axioms that are used by Haller [11] and Malawaski [21] to characterize the (non-efficient) Banzhaf value for TU-games.

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