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Potential optimality and robust optimality in multiattribute decision analysis with incomplete information: A comparative study

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ABSTRACT

The traditional approach for multiple attribute decision analysis with incomplete information on alternative values and attribute weights is to identify alternatives that are potentially optimal. However, the results of potential optimality analysis may be misleading as an alternative is evaluated under the best-case scenario of attribute weights only. Robust optimality analysis is a conservative approach that is concerned with an assured level of payoff for an alternative across all possible scenarios of weights. In this study, we introduce two measures of robust optimality that extend the robust optimality analysis approach and classify alternatives in consideration into three groups: strong robust optimal, weak robust optimal and robust non-optimal. Mathematical models are developed to compute these measures. It is claimed that robust optimality analysis and potential optimality analysis together provide a comprehensive picture of an alternative's variable payoff.

1. Introduction

In a multiattribute decision making (MADM) problem [5] the decision maker (DM) is faced with the task of identifying the best alternative in terms of a number of criteria (attributes). When precise information of the DM's preference is known the choice of the best alternative can be handled in a relatively straightforward manner, say, by evaluating the weighted sum of each alternative's values under all attributes as an aggregate value to gauge the alternative's payoff. However, in reality the DM is usually unable to elicit exact estimations of all decision parameters, such as alternative values and attribute weights [20]. Hence, multiattribute decision analysis with incomplete, or imprecise, or partial, information has become an important research direction in decision analysis.

Under incomplete information the exact parameters are not known but the constraints they satisfy, e.g., ordinal or interval judgments on them, might have been extracted. Various approaches to process incomplete information were proposed. Simulation studies showed that the rank order centroid weights and the maximum entropy weights are good approximates for ranked attribute weights [1,2,4]. Sarabando and Dias [15] developed centroid-based decision rules with a high likelihood of identifying the best alternative when both attribute weights and alternative values under every attribute are fully ranked.

Pairwise comparisons and potentially optimality analysis (PO analysis) are commonly used for incomplete information not restricted to the ordinal form [3,8,9,11–14]. While these approaches help narrow the DM's choices to a subset of the available alternatives, called nondominated (ND) alternatives and potentially optimal (PO) alternatives, the identification of the best alternative still eludes the DM unless there exists an alternative that outperforms its peers in all attributes. Furthermore, the quality of the alternatives recommended by these approaches is questionable. As Dias and Climaco [7] noted, the binary preference relations established on the basis of pairwise comparisons are not easy to utilize meaningfully. PO analysis assesses alternatives in their best-case scenarios only. Wang [19] demonstrated that choosing alternatives for PO may cause a significant loss if an unfavorable scenario occurs making the selected alternative severely inferior.

Given incomplete information on both alternative values and attribute weights, Park [13] introduced a three-level PO classification scheme: strong potentially optimal (strong PO), weak potentially optimal (weak PO) and potentially non-optimal (NPO). As summarized in Table 1, an alternative is strong PO if it is better than its peers for all possible scenarios of values and at least one vector of weights. An alternative is weak PO if it is the best for at least some scenarios of feasible values and weights. An alternative is NPO if it is not better than other alternatives under any scenario of decision parameters. Table 1 also presents robust optimality (RO), which will be detailed later in this paper.

In contrast to the optimistic attitude in PO analysis, the minmax regret analysis first proposed by Savage [16,17] is a conservative approach. The term “regret” means the discrepancy between the actual payoff of an alternative and the best one that could have been rendered with a different choice. Fishburn [9] suggested the use of the minmax regret criterion as a secondary principle for ranking PO

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Table 1
Optimality analysis.

		Imprecise weights	
		Best-case	Worst-case
Imprecise values	Best-case	Weak PO	Weak RO
	Worst-case	Strong PO	Strong RO

alternatives if an optimal alternative cannot be identified with the available information on weights (state probabilities). However, his exposition was limited to a few special forms of probability information. Wang [19] applied the minmax regret criterion to develop a RO analysis framework for MADM problems with incomplete information on both alternative values and attribute weights. The suggested approach ranks alternatives in terms of their regrets in the worst-case scenarios of unknown decision parameters.

It is noted that robustness has been defined and interpreted differently in the literature [6]. The maxmin criterion [10,18] captures the DM's aversion to the worst possible outcome of an alternative. However, it is generally believed that the minmax regret criterion adopted in the current study is not so extreme in its conservatism as the maxmin criterion [16]. In this study robustness is in the sense of immunizing an alternative's regret to the changes of attribute weights. Given exact values, an alternative is RO if it is better than its peers even in the worst-case scenario of weights. Compared to PO analysis, research on RO analysis is underdeveloped for MADM problems with incomplete information on both values and weights. Since RO analysis shall be attractive for a DM who has to make decisions under incomplete information, our study is motivated by the gap between its theoretical development and practical significance.

We present in this article two measures of RO and introduce RO definitions. As Table 1 indicates, under the worst-case scenario of weights, strong RO represents RO for *all* scenarios of values, while weak RO signifies RO for at least one scenario of values. This development facilitates the notion that incomplete information on values and weights leads to an alternative's variable optimality, quantified by its regret, for which the upper bound and lower bound are yielded by RO analysis and PO analysis, respectively. This study extends the RO analysis method Wang [19] proposed and greatly improves our understanding of optimality under incomplete information.

The remainder of the paper is organized as follows. In Section 2, we set the stage by overview the background of our study. In Section 3, RO analysis models are presented and RO classifications are introduced. We further discuss solving these models using mathematical programming techniques. In Section 4, a computational example is analyzed to illustrate RO analysis. Finally, concluding remarks are offered.

2. Background

Suppose that the DM evaluates a discrete set of alternatives $M = \{1,2,\cdots,m\}$ in terms of a set of attributes $N = \{1,2,\cdots,n\}$. The simplest and most common evaluation approach, namely the linear aggregation method, can be summarized as follows.

Let w_j be the weight of attribute $j \in N$ and x_{ij} be the value of alternative $i \in M$ with respect to attribute $j \in N$. It is assumed that the values x_{ij} associated with each attribute j are scaled to the interval [0,1], with 0 and 1 designating respectively the worst and best possible levels. Denote by $\mathbf{w} = (w_1,w_2,\cdots,w_n)^t$ and $\mathbf{x}_j = (x_{1j},x_{2j},\cdots,x_{mj})^t$, respectively, the vectors of attribute weights and values associated with each attribute $j \in N$. Given a n-tuple of value vectors $\mathbf{U} = (\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_n)$, we can construct $\mathbf{x}^i(\mathbf{U}) = (x_{i1},x_{i2},\cdots,x_{in})^t$, the vector of values for each alternative $i \in M$. When precise information is available, i.e., $\mathbf{w}$ and $\mathbf{U}$ are known exactly, the multiattribute value (MAV) of an alternative i is assessed using a linear additive weighting function $V_i(\mathbf{w}, \mathbf{U}) = \mathbf{w}^t\mathbf{x}^i(\mathbf{U})$.

It is more realistic to presume that the DM is able to obtain incomplete information on decision parameters only. Similar to Eum et al. [8] and Park [13], we assume that the weight vector $\mathbf{w}$ satisfies linear constraints, i.e., $\mathbf{w} \in S_\mathbf{w} = \{\mathbf{w}|\mathbf{A}\mathbf{w} \le \mathbf{a}\}$, where $S_\mathbf{w}$ is the set of all feasible weight vectors, $\mathbf{A}$ is a $p \times n$ matrix containing the constraint coefficients, p is the number of constraints and $\mathbf{a}$ is the vector of constraint right-hand-side values. Similarly, for each attribute $j \in N$ it is assumed that incomplete information on $\mathbf{x}_j$ leads to the set of feasible value vectors $S_{\mathbf{x}_j} = \{\mathbf{x}_j|\mathbf{B}_j\mathbf{x}_j \le \mathbf{b}_j\}$, where $\mathbf{B}_j$ is a $q_j \times m$ matrix containing constraint coefficients and $\mathbf{b}_j$ is the right-hand-side value vector with q_j elements. As Park [13] suggested, the constraints may have various configurations such as weak orders ($w_j \ge w_f$, $j,f \in N$ and $j \ne f$), interval estimates ($x_{ij}^L \le x_{ij} \le x_{ij}^U$, x_{ij}^L and x_{ij}^U are constants, $j \in N$ and $i \in M$) and ratio bounds ($v_{jf}^L \le w_j/w_f \le v_{jf}^U$, v_{jf}^L and v_{jf}^U are constants, $j,f \in N$) relations.

Let $S_\mathbf{U} = S_{\mathbf{x}_1} \times S_{\mathbf{x}_2} \times \cdots \times S_{\mathbf{x}_n}$ be the Cartesian product of the sets $S_{\mathbf{x}_j}$, $\forall j \in N$. Recall that given a tuple of value vectors $\mathbf{U} \in S_\mathbf{U}$, $\mathbf{x}^i(\mathbf{U})$ represents a feasible realization of the values of alternative i. We require that the sets $S_\mathbf{w}$ and $S_\mathbf{U}$ be not empty, i.e., the MADM problem is feasible. Not knowing exact parameters, the DM is prone to make a decision that could be proven "wrong" later, which would make the DM feel regret. The perspective of *regret* is adopted throughout this study.

A feasible scenario of decision parameters can be characterized by $(\mathbf{w}, \mathbf{U})$, where $\mathbf{w} \in S_\mathbf{w}$ and $\mathbf{U} \in S_\mathbf{U}$. Let ϕ be the set of all these feasible scenarios. The DM's regret associated with a scenario $(\mathbf{w}, \mathbf{U}) \in \phi$ for selecting alternative k, rather than alternative h, is triggered by the disparity between their actual payoffs. If we gauge the payoff of an alternative by its MAV, then the regret, denoted by $c_{kh}(\mathbf{w}, \mathbf{U})$, is calculated as $c_{kh}(\mathbf{w}, \mathbf{U}) = \mathbf{w}^t(\mathbf{x}^h(\mathbf{U}) - \mathbf{x}^k(\mathbf{U}))$.

We now present important concepts in the existing literature in terms of regret.

Definition 2.1. Alternative k is *dominated* by alternative h if $c_{kh}(\mathbf{w}, \mathbf{U}) > 0$ holds for any $\mathbf{w} \in S_\mathbf{w}$ and $\mathbf{U} \in S_\mathbf{U}$.

Definition 2.2. An alternative is *nondominated* if it is not dominated by any other alternative.

Definition 2.3. Alternative k is weak PO if $\alpha_k = \min_{(\mathbf{w},\,\mathbf{U}) \in \phi} \max_{h \in M} c_{kh}(\mathbf{w},\ \mathbf{U}) = 0$.

Definition 2.4. Alternative k is strong PO if $\beta_k = \max_{\mathbf{U} \in S_\mathbf{U}} \min_{\mathbf{w} \in S_\mathbf{w}} \max_{h \in M} c_{kh}(\mathbf{w},\ \mathbf{U}) = 0$.

For an overview of identifying dominance and potential optimality with incomplete information using mathematical programming techniques the reader is referred to [13].

Given a feasible scenario $(\mathbf{w}, \mathbf{U})$, $C_k(\mathbf{w}, \mathbf{U}) = \max_{h \in M} c_{kh}(\mathbf{w}, \mathbf{U})$ measures the loss that results from choosing alternative k without prior knowledge that $(\mathbf{w}, \mathbf{U})$ is the true scenario. It is evident that $C_k(\mathbf{w}, \mathbf{U}) \ge 0$, while $C_k(\mathbf{w}, \mathbf{U}) = 0$ indicates that alternative k is optimal under the scenario. By definition, an alternative is weak PO if it is optimal for *some* feasible scenarios of values and weights, while an alternative is strong PO if it is optimal for *all* feasible scenarios of values and at least one feasible vector of weights. Note $\beta_k \ge \alpha_k$. It follows that there exists at least one ND alternative and one weak PO alternative, but a strong PO alternative may not be available. We hence call an alternative k *strong potentially quasi optimal* (strong PQO) if $\beta_k = \min \beta_h$. As shown in Table 1, we can classify alternatives into three groups Using the two PO measures: strong PO (strong PQO), weak PO, and NPO.

PO analysis assesses an alternative's optimality in terms of the most favorable scenario of attribute weights. As Wang [19] demonstrated, adopting an alternative by its potentially optimality may eventually lead to a significant loss if the true scenario defies optimistic expectations. In contrast to PO analysis, minmax regret analysis is a conservative approach that advocates adopting an alternative even suboptimal in the best-case scenario of weights so as to minimize the worst-case