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Review article

# Genetic coding and united-hypercomplex systems in the models of algebraic biology

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ABSTRACT

Structured alphabets of DNA and RNA in their matrix form of representations are connected with Walsh functions and a new type of systems of multidimensional numbers. This type generalizes systems of complex numbers and hypercomplex numbers, which serve as the basis of mathematical natural sciences and many technologies. The new systems of multi-dimensional numbers have interesting mathematical properties and are called in a general case as "systems of united-hypercomplex numbers" (or briefly "U-hypercomplex numbers"). They can be widely used in models of multi-parametrical systems in the field of algebraic biology, artificial life, devices of biological inspired artificial intelligence, etc. In particular, an application of U-hypercomplex numbers reveals hidden properties of genetic alphabets under cyclic permutations in their doublets and triplets. A special attention is devoted to the author's hypothesis about a multi-linguistic in DNA-sequences in a relation with an ensemble of U-numerical sub-alphabets. Genetic multi-linguistic is considered as an important factor to provide noise-immunity properties of the multi-channel genetic coding. Our results attest to the conformity of the algebraic properties of the U-numerical systems with phenomenological properties of the DNA-alphabets and with the complementary device of the double DNA-helix. It seems that in the modeling field of algebraic biology the genetic-informational organization of living bodies can be considered as a set of united-hypercomplex numbers in some association with the famous slogan of Pythagoras "the numbers rule the world".

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## 1. Introduction

The main task of the mathematical natural sciences is the creation of mathematical models of natural systems. Development of models and formalized theories depends highly on those mathematical notions and instruments, on which they are based.

In the beginning of the XIX century the following opinion existed: the world possesses the single real geometry (Euclidean geometry) and the single arithmetic. But this opinion was neglected after the discovery of non-Euclidean geometries and of quaternions by Hamilton. Science understood that different natural systems could possess their own individual geometries and their own individual arithmetic (see this theme in the book (Kline, 1980)). The

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$$Z = a_0 \bullet 1 + a_1 \bullet i, \quad \text{or} \quad Z = a_0 \bullet \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_1 \bullet \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} a_0 & a_1 \\ -a_1 & a_0 \end{bmatrix}$$

| • | 1 | i |
|---|---|---|
| 1 | 1 | i |
| i | i | -1 |

**Fig 1.** Left: representations of complex numbers Z in its linear and matrix forms where the matrix [1,0; 0, 1] = E is the identity matrix and the matrix [0,1; −1, 0] represents the imaginary unit "i". Right: the multiplication table of basic elements of complex numbers.

example of Hamilton, who was unlucky during 10 years in his attempts to solve the task of description of transformations of 3D space by means of 3-dimensional algebras, is a very demonstrative one. This example says that, if a scientist does not guess correctly what types of numeric systems are adequate for the natural system investigated by him, he can waste many years without any result. One can add that effectiveness of analytical models of geometrical and physical-geometrical properties of separate natural systems (including laws of conservation, theories of oscillations and waves, theories of potentials and fields, etc.) depends on adequatecy of used numeric systems.

The idea about special mathematical peculiarities of living matter exists long ago. For example Vernadsky (1965) put forward the hypothesis on a non-Euclidean geometry of living nature but without any concrete definition of the type of such geometry. We also believe that living bodies live in their own bio-information space, which has specific algebraic and geometric properties. In our opinion many difficulties of modern science to understand genetic and other biological systems are determined by approaches to them from the non-adequate algebraic standpoints, which were developed formerly for other systems.

But how one can find appropriate algebraic approaches in biology if various species of organisms differ from each other so significantly in their morphogenetic and many other features? The discovery of the genetic code, basic features of which are general for all biological organisms, has allowed hoping that such mathematical problem can be solved by means of investigation of genetic code structures. It seems an important task to investigate from different standpoints what systems of multi-dimensional numbers are connected or can be connected with ensembles of parameters of the genetic code and with relevant bio-information spaces. Some results of such investigation are presented in this article.

Various kinds of complex and hypercomplex numbers – complex numbers, double (or hyperbolic) numbers, quaternions and others – are used in different branches of modern science. They have played the role of the magic tool for development of theories and calculations in problems of heat, light, sounds, fluctuations, elasticity, gravitation, magnetism, electricity, current of liquids, quantum-mechanical phenomena, special theory of relativity, nuclear physics, etc. For example, in physics thousands of works – only in XX century – were devoted to quaternions of Hamilton (their bibliography is in (Gsponer and Hurni, 2008)).

Each kind of hypercomplex numbers has its own multiplication table of basic elements. As known, complex and hypercomplex numbers can be written not only in linear forms but also in matrix forms of their representation (in cases of associative algebras).

Hypercomplex numbers are a generalization of complex numbers. The latter have one real unit "1" and one imaginary unit "i" with the property $i^2 = -1$. Complex numbers are written in a linear form and also in a matrix form (Fig. 1).

As known, hypercomplex numbers have a single real unit and additional quantity of imaginary units, all of which in the total are called basic elements of hypercomplex numbers. The expression (1) shows their linear form of writing in general case:

$$a_0E + a_1e_1 + a_2e_2 + \ldots\ldots + a_ne_n \tag{1}$$

where E denotes the real unit (or the identity matrix E in the case of the matrix representation of hypercomplex numbers), $e_1, e_2, \ldots, e_n$ denote imaginary units, $a_0, a_1, \ldots, a_n$ denote real coefficients. The set of these basic elements ($E, e_1, e_2, \ldots, e_n$) should be closed under multiplication. This means that multiplication of any two basic elements with each other always gives again one of these elements or their linear superposition (Kantor and Solodovnikov, 1989). The results of the multiplication of the basic elements are represented in the corresponding multiplication table, which is individual for each type of hypercomplex numbers. If a system (1) does not possess the real unit E, which is mutual for all terms in the expression (1), it is not a system of hypercomplex numbers.

In the result of our study of systems of structured alphabets of DNA and RNA in matrix forms of their representations (Petoukhov, 2008, 2012, 2014, 2016a,b, 2017; Petoukhov and He, 2010; Petoukhov and Petukhova, 2017a,b), we have revealed that these alphabetic structures are connected with special $(2^n \times 2^n)$-matrix operators. Various decompositions of these operators show their connection with special $2^n$-dimensional numeric systems with the following features:

1) they are not hypercomplex numbers in the whole because of absence of the identity matrix E in their decompositions (that is the general real unit is absent among basic elements of such systems);
2) they contain two or more blocks, each of whose is a sparse matrix and represents complex numbers or hypercomplex numbers. Each of blocks contains an individual set of basic elements, which play role of its own real unit and imaginary units but only inside this separate block; by this reason we call them as local-real units and local-imaginary units.

In other words, two or more different systems of complex or hypercomplex numbers exist in this case not separately but are combined together into a single structure by means of a summation of sparse matrices. The author calls such multi-dimensional numeric systems as "multi-block united-hypercomplex systems" (or more briefly, "united-hypercompex numbers" or simply "U-numbers"). Such systems of united-hypercomplex numbers are

$$H = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad H^{(2)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \qquad H^{(3)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

**Fig. 2.** The beginning of the tensor (or Kronecker) family of Hadamard matrices $H^{(n)}$, where (n) means a tensor power. Black cells correspond to components "+1", white cells – to "-1".

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