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# Effect of diseases on symbiotic systems

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ABSTRACT

There are many species living in symbiotic communities. In this study, we analyzed models in which populations are in the mutualism symbiotic relations subject to a disease spreading among one of the species. The main goal is the characterization of symbiotic relations of coexisting species through their mutual influences on their respective carrying capacities, taking into account that this influence can be quite strong. The functional dependence of the carrying capacities reflects the fact that the correlations between populations cannot be realized merely through direct interactions, as in the usual predator-prey Lotka-Volterra model, but also through the influence of each species on the carrying capacities of the other one. Equilibria are analyzed for feasibility and stability, substantiated via numerical simulations, and global sensitivity analysis identifies the important parameters having a significant impact on the model dynamics. The infective growth rate and the disease-related mortality rate may alter the stability behavior of the system. Our results show that introducing a symbiotic species is a plausible way to control the disease in the population.

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## 1. Introduction

Symbiosis is a phenomenon for which mutual associations of different living organisms benefit from each other. It may be fundamental for system stability (Boucher et al., 1982; Thompson, 1994; Bronstein et al., 2001). Symbiotic relationships appear to be very common in biological and ecological communities (Turnbaugh et al., 2007; Nelson, 1993). In ecology, symbiosis can be found for instance in various associations between, e.g. plant roots and fungi, coral organisms and various types of algae, cattle egrets and cattle, mangroves and root borers, spiders and parasitic wasps, invertebrates and their epibionts, corals and fish and also among the diatom mats in the ocean (Boucher, 1988; deLaplante et al., 2011; Richerson and Boyd, 1998; Ahmadjian and Paracer, 2000; Townsend et al., 2002; Yukalov et al., 2012a; Venturino, 2007). The concept of mutual help between different communities can be extended beyond the purely biological realm, to encompass also phenomena of sociological, economical and financial type (von Hippel, 1988, 2005; Graedel and Allenby, 2003; Muller and Krauss, 2005; Pedruzzi et al., 2016; Goff, 2011; Yukalov et al., 2012b).

In biology, three different symbiotic associations are generally identified (Boucher, 1988; deLaplante et al., 2011; Richerson and Boyd, 1998; Ahmadjian and Paracer, 2000; Townsend et al., 2002). In *mutualism*, all the different species benefit from the interactions. In *parasitism*, the parasite get benefits at the expenses of the host; for instance bacteria, helmints and viruses fall generally in this category. Finally, *commensalism* denotes the interaction for which one organism benefits but for the other one there is no gain or loss; for example, tiger and golden jackals, who link themselves to a tiger maintaining a safe distance, to feed on the remnants of the tiger's prey. Commensalism represents a marginal relationship with respect to the other two relationships, a kind of in-between state (Haque and Venturino, 2009). Symbiotic associations have also been considered within the larger situations of food webs, where some of the other populations are in competition with each other (Kooi et al., 2004; Kumar and Freedman, 1989; Ishikawa, 1988; NIH, 2012; Zaghrout, 1991; Grossman and Helpman, 1991; Finkes et al., 2006; Caccherano et al., 2012).

In ecoepidemiology, the effect of epidemics on the underlying demographic populations interactions are studied (Gyllenberg et al., 2006; Venturino, 1994, 1995; Chattopadhyay and Arino, 1999). Mainly, ecoepidemic systems based on predator-prey or competing demographics have been investigated (Chattopadhyay and Bairagi, 2001; Venturino, 2001; Chattopadhyay and Pal, 2002; Chattopadhyay et al., 2003; Kumar and Freedman, 2002). But also

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examples of symbiotic associations affected by disease exist in nature, e.g. several mushrooms such as *Cantharellus cibarius*, *Boletus* spp., *Amanita* spp. etc., with chestnut trees (*Castanea sativa*), affected by the disease chestnut cancer (*Endothia parasitica*). Also, symbiotic interactions are found just among bacteria (Zhao and Lin, 2005), or among bacteria and other organisms, plants, plants and mushrooms (Boursaux-Eude and Gross, 2000; Sapp, 1994; NIH, 2012; Marino et al., 2008) with effects that reach their whole ecosystem (Douglas, 1994). Based on these facts, symbiotic situations encompassing diseases have been considered first in Venturino (2007); Hadeler and Freedman (1989) and then in Bosica et al. (2014), where different strains have been investigated.

In symbiotic relations, the coexisting species interact with each other by mainly affecting the carrying capacities of each other (Boucher, 1988; deLaplante et al., 2011; Richerson and Boyd, 1998; Ahmadjian and Paracer, 2000; Townsend et al., 2002). For instance, humans have increased the carrying capacity of a few other species, the domestic animals, to exploit their resources, e.g. cows, horses, pigs, sheep, goats, dogs, cats, and chickens. Also, in agriculture plants such as wheat, rice, barley, maize, tomato, and cabbage have benefited by the human intervention (Begon et al., 1986). Thus, the carrying capacities of symbiotic species can be thought as variable quantities, functions of the population sizes (Stewart and Cavanaugh, 2006; Gan and Lin, 2008), in general assumed in the form of polynomials of the population density (Richard, 1993; Yukalov et al., 2012a,b). If the carrying capacity is represented by a linear functional of the other population density only, then this type of symbiosis can be termed passive weak, since the species do not directly interact in the process of varying the livelihoods of their neighbors, in the sense that the impacts of species abundance on carrying capacities are linear (Yukalov et al., 2014). For instance, most land ecosystems rely on symbiosis between the plants that extract carbon from the air and mycorrhizal fungi extracting minerals from the ground. On the other hand, if in the process of influencing the livelihoods of each other the species directly interact, then their carrying capacities are approximated by the bilinear expressions assuming that their interactions are sufficiently weak (Yukalov et al., 2014). This type of symbiosis can be termed active weak. Examples of this type of symbiosis taken from economics or the human world could be the relations between different firms producing goods in close collaboration with each other, or the relation between basic and applied sciences.

In the present study, we consider two populations, which are in symbiotic relationship. Further, one of the two populations is suffering from a recoverable disease. The aim of this article is mainly to assess how the system behavior changes in the presence of a disease in a single population and what is the epidemics effect on the other, disease-unaffected, population. Also, we would like to know whether mutualism helps to eradicate the disease from the infected population.

The above ecological questions are answered by the model analysis in two different situations. First, we consider the model with passive weak symbiosis, where the carrying capacity is represented by a linear functional of the other population density only (Yukalov et al., 2014). Secondly, we study the model with active weak symbiosis, where the carrying capacity depends bilinearly on both symbiotic populations (Yukalov et al., 2014). A schematic diagram depicting the dynamics of the proposed models is shown in Fig. 1.

Rest of the paper is organized as follows: in the next section, we introduce the model for the passive weak symbiotic relation and analyze its equilibria for feasibility and stability in Section 3. In Section 4, we report the related numerical simulations. Global sensitivity analysis is performed in Section 5. Further, in Section 6, we introduce the active weak symbiosis model and analyze it mathematically in the next section and numerically in Section 8. Section 9 contains a final discussion of the findings.

## 2. Model with passive weak symbiosis

As we already discussed, we divide our study in two cases: firstly, we considered **passive weak symbiotic relations** between two populations and secondly, **active weak symbiotic relation**. To distinguish the model variables and parameters, we have used the **superscript** '*p*' in the notations for the model with passive weak symbiotic relations and the **superscript** '*a*' in the notations for the model with active weak symbiotic relations.

Let $S^p$ and $P^p$ be two populations in passive weak symbiotic relation. We assume that a recoverable disease spreads by contact among the $S^p$ populations, giving rise to infected individuals $I^p$. Let $r_1^p, r_2^p$ and $r_3^p$ be the intrinsic growth rates of $S^p$, $I^p$ and $P^p$ populations, respectively. It is assumed that the infectives reproduce at different rate than the susceptibles, producing offsprings that are themselves infected. Thus the disease is vertically transmitted. Further, it also spreads horizontally among the individuals at rate $\lambda^p$. The infected individuals recover from the disease at rate $\nu^p$, re-entering into the susceptible class. The infected individuals also experience disease related mortality at the rate $\mu^p$. Let $K_1^p$ denote the carrying capacity of $S^p$ and $I^p$ populations. Because of the positive interactions among the species, the population $P^p$ increases the carrying capacity of $S^p$ and $I^p$ at the rates $a_1^p$ and $a_2^p$, respectively. A similar effect is produced by the $S^p$ and $I^p$ population on the carrying capacity $K_2^p$ of the $P^p$, at rates $b_1^p$ and $b_2^p$, respectively. Let $a_{11}^p$ and $a_{22}^p$ denote the intraspecific competitions for $S^p$ and $I^p$ populations. Similarly, let $a_{12}^p$ and $a_{21}^p$ represent the interspecific competitions for the corresponding populations. The parameters $a_1^p, a_2^p, b_1^p$ and $b_2^p$ characterize the productive or destructive influence of the relevant species on its counterpart. With these assumptions, the model, in which all parameters are assumed to be nonnegative, reads

$$\begin{aligned}
\frac{dS^p}{dt} &= r_1^p S^p \left(1 - \frac{a_{11}^p S^p + a_{12}^p I^p}{K_1^p + a_1^p P^p}\right) - \lambda^p S^p I^p + \nu^p I^p, \\
\frac{dI^p}{dt} &= r_2^p I^p \left(1 - \frac{a_{21}^p S^p + a_{22}^p I^p}{K_1^p + a_2^p P^p}\right) + \lambda^p S^p I^p - \mu^p I^p - \nu^p I^p, \\
\frac{dP^p}{dt} &= r_3^p P^p \left(1 - \frac{P^p}{K_2^p + b_1^p S^p + b_2^p I^p}\right).
\end{aligned} \tag{2.1}$$

## 3. Mathematical analysis

### 3.1. Equilibria

The system's equilibria are the following points.

The ecosystem's extinction equilibrium $E_0^p(0, 0, 0)$, always feasible.

The disease-free equilibrium with only the first population, which is also always feasible $E_1^p\left(K_1^p (a_{11}^p)^{-1}, 0, 0\right)$.

The equilibrium with the second population only, $E_2^p(0, 0, K_2^p)$, again always feasible.

The disease-free equilibrium $E_3^p(S_3^p, 0, P_3^p)$, with

$$S_3^p = \frac{K_1^p + a_1^p K_2^p}{a_{11}^p - a_1^p b_1^p}, \quad P_3^p = \frac{b_1^p K_1^p + a_{11}^p K_2^p}{a_{11}^p - a_1^p b_1^p}.$$

$E_3^p$ is feasible if

$$a_{11}^p - a_1^p b_1^p > 0. \tag{3.1}$$

In the absence of infective population $I^p$, the condition (3.1) has a geometrical interpretation as follows. Solving the first and the

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