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# Coalition formation based on marginal contributions and the Markov process

Stephen Shaoyi Liao [a,*], Jia-Dong Zhang [b], Raymond Lau [a], Tianying Wu [a]

[a] Department of Information Systems, City University of Hong Kong, Hong Kong
[b] Department of Computer Science, City University of Hong Kong, Hong Kong

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ABSTRACT

With competition intensifying in the globalized economy, an increasing number of firms are forming coalitions or alliances to improve purchasing efficiency and reduce operating costs in various industries. Forming such coalitions or alliances has become a key research challenge in two important kinds of decision support systems, namely group support systems and negotiation support systems, since the number of possible coalitions is very large in most cases. Most of the existing research on coalition formation focuses on generation of optimal structures alone. Nevertheless, self-interested agents, who are mainly concerned with their own benefits, usually determine whether to join a coalition on the basis of payoffs they can possibly get from the coalition. Accordingly, in this paper, we propose a novel method of coalition formation to enable agents to improve their own benefits based on marginal contributions and the Markov process. Our method considers both coalition structure generation and payoff division which are two primary concerns of group and negotiation support systems.

By using a real-world scenario, we give an example of formation of retailer coalitions to illustrate the proposed method. Finally, it is experimentally showed that the method proposed in this paper is effective and efficient, compared with other existing methods. The coalitions generated by our algorithms can significantly increase most agents' payoffs. The managerial implication of our research is that firms can apply the proposed method to identify the most beneficial coalition network with their business partners.



## 1. Introduction

Coalition formation in the cooperative game theory is an important step that can help improve social welfare and individual agents' interests in two important kinds of decision support systems, namely group support systems and negotiation support systems [4]. It has been studied by many researchers from different perspectives with different methods [2,27–29,32]. Coalition formation is always a big challenge given the strategic conflicts among the participating agents in group decision-making or negotiation processes within a wide variety of political, economic, and social settings [12,15]. Coalition formation process involves three main activities, that is, coalition structure generation, payoff calculation and payoff division among members. While payoff division and coalition structure generation are important issues that need to be studied independently [5,27,28], it is desirable to extend those studies by focusing on both simultaneously in the context of self-interested agents [30]. Most of the existing research works have considered only the coalition structure generation when dealing with the coalition formation problem [30,27,29].

A coalition $C$ is a subset of agent set $N$, $N=\{1, \ldots, n\}$. Such a coalition can improve the performance of individual agents and/or the system as a whole, especially when tasks cannot be performed by a single agent or when a group of agents performs the tasks more efficiently [26]. A value $v(C)$, called the payoff in this paper, indicates how beneficial coalition $C$ would be if it was formed and $v$ is called the characteristic function. The coalition structure, usually denoted by $CS$, is a partition of the agent set. Dealing with the problem of coalition formation, most of the extant research has considered the value of the coalition structure $CS$ as $V(CS) = \Sigma_{C \in CS} v(C)$ and coalition structure $CS^*$ where $V(CS^*) = \text{argmax}\, \Sigma_{C \in CS} v(C)$ as the optimal one.

Obviously, such a viewpoint is reasonable when agents in the game are cooperative. What concerns the agents is maximization of social welfare while payoff division is a non-issue. However, what concerns self-interested agents is how to enhance their own incomes. A coalition may earn a good payoff as a whole but a self-interested agent may not participate in it if the payoff division is not satisfactory to that particular agent. Instead, the agent may join another coalition that might be earning a smaller aggregate payoff but the agent can get a higher payoff. Thus we consider coalition formation from the perspective of individual agents. In this paper, we call coalitions formed by our method individual agents' benefits based coalitions ($ICs$) and the corresponding coalition structure individual preference based coalition structure ($ICS$) (refer to Definition 3.4).

Against this background, we construct an integrated theory that encompasses coalition structure generation and payoff division where we suppose $v$ is known during the model description process. This

* Corresponding author.
*E-mail addresses:* issliao@cityu.edu.hk (S.S. Liao), jzhang26-c@my.cityu.edu.hk (J.-D. Zhang).

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paper discusses coalition formation from the perspective of individual agents based on marginal contributions and the Markov process. Whether a coalition is formed depends on payoffs member agents can get from the coalition. The Shapley value [31], as a method of payoff division, is always non-empty and unique in all types of cooperative games. On the other hand, the Shapley value reflects the fairness well since individual agents get their payoffs according to their contributions to the corresponding coalition. Therefore, we take the Shapley value as the payoff division method in this paper. Agents choose coalitions from which they can get higher Shapley values.

In this paper, against the exponential growth of coalition structure space $O\ (n^n)$ with agents' number $n$ [28], a method of pruning the search space is given at first. The method involves forming superior coalitions and discarding strictly inferior coalitions simultaneously, based on the verified relationship between Shapley values and marginal contributions. Theoretically, this method reduces the search space drastically and the search space is pruned to $O\ (2^{n-1})$ in the worst case. Experimental analyses show that the search space is reduced by more than 99.9% when the number of agents is more than 7.

The following endeavor is to find the optimal coalition structure (namely, *ICS*) from the pruned search space. It is still a complex task to solve the *ICS* from the perspective of individual agents even though the search space has been narrowed drastically. Fortunately, the process of individual agents' transitions among different coalition structures is a typical Markov process, where a current state transits into future states with certain distribution probabilities [16]. So we model individual agents' transitions among the coalition structures by the Markov process and search the *ICS* based on probability distribution of Markov states. However, the typical way of figuring out Markov states' probability distribution is random sampling, which is complex and time-consuming [17,19]. Fortunately, the Markov chain modeled in this paper has some excellent properties that allow us to obtain the probability distribution by solving the stationary distribution vector.

The number of firms considering coalitions to enhance competitiveness and reduce operating costs is growing [20,13]. The illustrative example given involves a one-supplier several-retailers' two-echelon supply chain where retailers form coalitions (called retailer-coalitions) to buy products from the supplier. Numerical analyses show that coalitions increase most retailers' payoffs.

Moreover, analyses of results of simulation experiments in the context of the retailer-coalition show that the search space is cut down to less than 0.1% when there are more than 7 agents in the cooperative game. Comparison of running time of different coalition formation ways shown in Table 3 indicates that the method proposed in this paper can be carried out efficiently. Moreover, from the payoff divisions shown in Fig. 5 we can see that the formed *ICs* can increase agents' incomes remarkably.

Importantly, group and negotiation support systems can leverage our novel computational method to generate effective coalition structures and maximize individual agents' benefits during group decision-making or negotiation processes. Moreover, our efficient method can also help develop practical solutions given the complexity of multi-agent negotiations [10,14]. For instance, in the context of group-buying, the complexity of the problem space involved may well exceed a buyers capacity and capability of processing information related to multiple parties in a timely manner. Fortunately, our proposed method enables buyers to efficiently determine the optimal buying groups so that they are able to maximize their total surplus, as illustrated by a real-world example in later section.

The main contributions of this paper can be summarized as follows:

- First of all, we prune search space of coalition structures by forming superior coalitions and discarding strictly inferior coalitions simultaneously, based on the verified relationship between Shapley values and marginal contributions (Section 3).
- Further, we identify and show some important properties of the Markov process for modeling individual agents' transitions among the coalition structures. These properties enable us to efficiently and correctly solve the probability distribution of Markov states and then find the optimal coalition structure, i.e. the individual preference based coalition structure (*ICS*) (Section 4).
- Finally, the proposed method is illustrated by a real world example and is experimentally evaluated by comparing it with three benchmark methods, which shows the efficiency and effectiveness of our method (Sections 5, 6).

The rest of the paper is structured as follows. Section 2 describes some related works. Section 3 introduces the algorithm for pruning search space based on the concept of marginal contributions. Section 4 models individual agents' transition process among coalition structures by the Markov process and finds the *ICs* by figuring out the probability distribution of Markov states. Section 5 gives a numerical example of a retailer-coalition. Section 6 describes experimental analyses and Section 7 concludes.

## 2. Related work

Coalition formation has received considerable attention in recent years. As mentioned in Section 1, the coalition formation process mainly includes three activities [29]: calculation of coalitions' payoffs, generation of the coalition structure, and division of the payoffs.

These three activities interact. For example, the coalition that an agent wants to join depends on the portion of the payoff that the agent would get. However, in the long run it would be desirable to construct an integrated theory that encompasses all three activities [30].

Most of the existing work concentrates on coalition structure generation only when considering the problem of forming coalitions [30,28,27,29,32]. The researchers take coalitions in the coalition structure $CS^*$, where $V(CS^*) = \text{argmax}\ \Sigma_{C\ \in\ CS}v(C)$, as the optimal coalitions. A lot of algorithms have been developed from this viewpoint, including the improved dynamic programming algorithm (IDP) [27], heuristic algorithms [32], anytime optimal algorithms (IP-Uniform and IP-Normal) [30,29] and Markov-based algorithms [17]. The Markov based algorithms have three advantages in comparison to the others. First, the coalition structure in each period of the Markov chain is endogenously determined, which allows us to study how coalitions evolve over time [3]. Second, the conflicts among agents are solved and the agents are grouped into coalitions according to their satisfaction degrees [17]. Third, the Markov chain combines two questions of stability with explicit monitoring of coalition formation [6].

Nevertheless, payoff division is a significant activity since all that the agents want to do is to improve their own incomes, i.e. agents are self-interested. Fortunately, a variety of methods have been developed to divide payoffs [8,9,14,2]. We compare two most well-known methods: the Shapley value [31,18] and the Banzhaf value [1,22] to emphasize the strengths of the Shapley value. First, the Shapley value is perfectly consistent and theoretically elegant but the Banzhaf value is a mere gimmick with no coherent theoretical underpinnings. Second, the Banzhaf value violates equiprobability in a very queer way and, as a result, a certain coalition is vulnerable to the defection of only one particular member [8]. Finally, the Shapley value possesses many excellent properties such as existence, uniqueness, fairness, monotonicity and practical usefulness. Several applications use Shapley values for payoff division [5,11]. In particular, [11] analyzed payoff division by using Shapley value in the context of "centralizing inventory in supply chains". In their work, coalition structure generation is not considered with the assumption that the supply chain has been formed. However, just considering the payoff division is not sufficient in most of the scenarios since discussing payoff division before knowing what coalitions are formed is meaningless.