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An approach to avoiding rank reversal in AHP☆

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Abstract

Analytic hierarchy process (AHP) has been considerably criticized for possible rank reversal phenomenon caused by the addition or deletion of an alternative. This paper looks into the cause of rank reversal phenomenon and finds that rank reversal is caused by change of local priorities before and after an alternative is added or deleted. An approach is therefore proposed to keep the local priorities unchanged to avoid rank reversal phenomenon. Two well-known numerical examples are re-examined using the proposed approach to demonstrate its validity and practicability in rank preservation.

Keywords: Analytic hierarchy process; Multiple criteria decision making; Rank reversal; Rank preservation

1. Introduction

Analytic hierarchy process (AHP), as a very popular multiple criteria decision making (MCDM) tool, has been considerably criticized for its possible rank reversal phenomenon, which means changes of the relative rankings of the other alternatives after an alternative is added or deleted. Such a phenomenon was first noticed and pointed out by Belton and Gear [3], which leads to a long-lasting debate about the validity of AHP [5,6,8–10,13,15,18,19,23–25,29,31,32,34,35], especially about the legitimacy of rank reversal [7,12,16,17,21,26].

In order to avoid the rank reversal, Belton and Gear [3] suggested normalizing the eigenvector weights of alternatives using their maximum rather than their sum, which was usually called B–G modified AHP. Saaty and Vargas [21] provided a counterexample to show that B–G modified AHP was also subject to rank reversal. Belton and Gear [4] argued that their procedure was misunderstood and insisted that their approach would not result in any rank reversal if criteria weights were changed accordingly. Schoner and Wedley [25] presented a referenced AHP to avoid rank reversal phenomenon, which requires the modification of criteria weights when an alternative is added or deleted. Schoner et al. [27] also suggested a method of normalization to the minimum and a linking pin AHP (see also [28]), in which one of the alternatives under each criterion is chosen as the link for criteria comparisons and the values in the linking cells are assigned a value of one,

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with proportional values in the other cells. Barzilai and Golany [1] showed that no normalization could prevent rank reversal and suggested a multiplicative aggregation rule, which replaces normalized weight vectors with weight–ratio matrices, to avoid rank reversal. Lootsma [11] and Barzilai and Lootsma [2] suggested a multiplicative AHP for rank preservation. Vargas [33] provided a practical counterexample to show the invalidity of the multiplicative AHP. Triantaphyllou [30] offered two new cases to demonstrate that the rank reversals do not occur with the multiplicative AHP, but do occur with the AHP and some of its additive variants. Leung and Cao [10] showed that Sinarchy, a particular form of analytic network process (ANP), could prevent rank reversal. As an integrative view, the AHP now supports four modes, called Absolute, Distributive, Ideal and Supermatrix modes, respectively, for scaling weights to rank alternatives [12,15,19,22]. In absolute mode, alternatives are rated one at a time and there is no rank reversal when new alternatives are added or removed. The distributive mode normalizes alternative weights under each criterion so that they sum to one, which does not preserve rank. The ideal mode preserves rank by dividing the weight of each alternative only by the weight of the best alternative under each criterion. The supermatrix mode allows one to consider dependencies between different levels of a feedback network. More recently, Ramanathan [14] suggested a DEAHP, which is claimed to have no rank reversal phenomenon. But in fact, it still suffers from rank reversal.

Our literature review shows that the rank reversal phenomenon has not been perfectly resolved and there still exist debates about the ways of avoiding rank reversals. So, this paper looks again into the cause of rank reversal and offers an alternative approach to avoid rank reversal.

The paper is organized as follows. In Section 2, we examine the rank reversal phenomenon using the numerical examples provided by Belton and Gear [3] and Saaty and Vargas [21]. In Section 3, we analyse the cause of rank reversal and propose an approach to avoid it. The two numerical examples are re-examined using the proposed approach to verify its validity and practicability in rank preservation. The paper is concluded in Section 4.

2. The rank reversal phenomenon

Belton and Gear [3] demonstrated the rank reversal phenomenon in AHP using a numerical example, which involves three consistent comparison matrices over four alternatives A, B, C and D with respect to three criteria *a*, *b* and *c*, respectively, where D is a copy of B and the three criteria were assumed to be equally important. They first took no account of the alternative D and derived a ranking for A, B and C, and then considered the four alternatives together and derived a ranking for A, B, C and D, only to find that the ranking between A and B was reversed after the alternative D was added. Tables 1 and 2 show the local and composite weights for the alternatives before and after the addition of D.

As can been seen from Table 2, the ranking between A and B is B≻A before D is introduced, but becomes A≻B after D is added. The ranking is reversed. Such a phenomenon is referred to as rank reversal, which may occur not only when an alternative is added, but also when an alternative is removed. Belton and Gear thought the reason for rank reversal to happen was due to improper normalization, which normalizes the weights of alternatives to sum to one. To avoid the rank reversal, they suggested normalizing the weights of alternatives using their maximum rather than their sum. That is to say, the weights of alternatives under each criterion should be divided by their maximum, which can be expressed as:

$$\bar{w}_i = \frac{w_i}{\max\limits_{k\in\{1,\ldots,n\}}\{w_k\}}, \qquad i = 1,\ldots,n \tag{1}$$

where $W=(w_1, \ldots, w_n)^T$ is eigenvector weight vector and $\bar{W}=(\bar{w}_1, \ldots, \bar{w}_n)^T$ is B–G normalized weight vector,

Table 1
Comparison matrices and the local weights for alternatives A, B, C and D under three criteria

Criterion (importance)	Alternatives	A	B	C	D	EM weights	BG weights
Criterion *a* (1/3)	A	1	1/9	1		1/11	1/9
	B	9	1	9		9/11	1
	C	1	1/9	1		1/11	1/9
Criterion *b* (1/3)	A	1	9	9		9/11	1
	B	1/9	1	1		1/11	1/9
	C	1/9	1	1		1/11	1/9
Criterion *c* (1/3)	A	1	8/9	8		8/18	8/9
	B	9/8	1	9		9/18	1
	C	1/8	1/9	1		1/18	1/9
Criterion *a* (1/3)	A	1	1/9	1	1/9	1/20	1/9
	B	9	1	9	1	9/20	1
	C	1	1/9	1	1/9	1/20	1/9
	D	9	1	9	1	9/20	1
Criterion *b* (1/3)	A	1	9	9	9	9/12	1
	B	1/9	1	1	1	1/12	1/9
	C	1/9	1	1	1	1/12	1/9
	D	1/9	1	1	1	1/12	1/9
Criterion *c* (1/3)	A	1	8/9	8	8/9	8/27	8/9
	B	9/8	1	9	1	9/27	1
	C	1/8	1/9	1	1/9	1/27	1/9
	D	9/8	1	9	1	9/27	1