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# Robust ordinal regression for multiple criteria group decision: UTA$^{GMS}$-GROUP and UTADIS$^{GMS}$-GROUP

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ABSTRACT

We introduce the principle of robust ordinal regression to multiple criteria group decision, and we present two new methods using a set of additive value functions as a preference model, called UTA$^{GMS}$-GROUP and UTADIS$^{GMS}$-GROUP. With respect to the set of decision makers (DMs), we consider two levels of certainty for the results. The first level is related to the necessary or possible consequences of indirect preference information provided by each DM, whereas the other refers to the subset of DMs agreeing for a specific outcome. In this way, we investigate spaces of consensus and disagreement between the DMs. The proposed methods are illustrated by examples showing how they can support real-world group decision.


## 1. Introduction

*Multiple criteria aggregation model* aims at aggregating vector evaluations of alternatives in a way consistent with the value system of the decision maker (DM). It induces a preference structure in a set of alternatives $A$, and, therefore, it is also called *preference model*. Its subsequent proper exploitation permits to arrive at a final recommendation, which is proposed to the DM. In this paper, preferences of the DMs on a set of alternatives will be modeled with the use of the Multi-Attribute Utility Theory (MAUT) [18]. The purpose of MAUT is to represent these preferences by an overall value (utility) function $U(a)=U(g_1(a),\ldots,\ g_m(a))$: $\mathcal{R}^m\rightarrow\mathcal{R}$. The comprehensive value of an alternative serves as an index used to decide the position in the ranking, or presence in the subset of the best alternatives, or the assignment into one of predefined and ordered classes. The simplest form of the value function is the additive form. It is important to stress that its use involves compensation between criteria, which are all reduced and expressed in the same unit, and requires rather strong assumption about mutual independence in the sense of preference, which is often difficult to met (see [5,18]). However, as noted in [25], these requirements do not pose significant problems in a posteriori analysis. Moreover, additive value functions are appreciated by the MCDA community for an easy interpretation of numerical scores of alternatives, as well as for possibility of aggregating quantitative and qualitative evaluations.

Using additive value functions requires specification of the parameters related to the formulation of marginal value functions $u_j(g_j(a))$, $j=1,\ldots,m$. These parameters follow either directly or indirectly from preference information provided by the DM. Recently, MCDA methods based on indirect preference information and on the *disaggregation–aggregation (or regression) paradigm* [14] are considered more interesting. It is the case, because they require less cognitive effort from the DM in answering questions concerning her/his preferences. The philosophy underlying the disaggregation–aggregation paradigm is to find a mathematical model able to reproduce exemplary decisions of the DM. Precisely, the DM provides some holistic judgments on a set of reference alternatives $A^R\subseteq A$, and from this information the parameters of a decision model are induced using a methodology called *ordinal regression* (see [26]). The ordinal regression consists in the resolution of mathematical programs in order to infer compatible instances of a considered preference model, which restore the exemplary decisions for reference alternatives. It has been used for at least fifty years in the field of multidimensional analysis. Historically, it has been first applied within MAUT to assess weights of an additive linear value function [27], and then to assess parameters of an additive piece-wise linear value

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function [13]. The latter method, called UTA, initiated a stream of further developments, in both theory and applications [25].

We say that an instance of a preference model is *compatible* with preference information given by the DM, if it is able to restore her/his holistic judgments. Usually, among many consistent instances of a preference model, only one specific instance is considered to give a recommendation. Since its choice is rather difficult and arbitrary to a large extent, *robust ordinal regression* has been proposed recently with the aim of taking into account *all* compatible instances of a preference models [9]. The first robust ordinal regression method has been the generalization of the UTA method, called UTA$^{GMS}$ [6]. In UTA$^{GMS}$, instead of only one compatible additive value function composed of piecewise-linear marginal functions, all compatible additive value functions composed of general monotonic marginal value functions are taken into account. Further, this approach has been extended in the UTADIS$^{GMS}$ method to deal with sorting problems [8], and in ELECTRE$^{GKMS}$, which is a general scheme implementing robust ordinal regression to outranking methods [10]. Robust ordinal regression has also been applied to preference model based on Choquet integral in order to handle interaction among criteria [1].

The family of methods based on robust ordinal regression has been originally designed to deal with preferences expressed by a single DM. However, it is *group decision-making* that is among the most important and frequently encountered processes within companies and organizations [3,29,31]. Typical examples of such problems can be found in management and business, e.g., evaluation of consumer preferences, personnel selection, or allocation of priorities to projects (see, e.g., [12]).

In this paper, we present in detail the principle of robust ordinal regression for group decision. Its first general idea has been introduced in [7]. Precisely, we consider the multiple criteria decision methods to which robust ordinal regression has been originally applied, and we propose corresponding methods which deal with preferences expressed by a set of DMs. We focus on methods employing a set of additive value functions as the preference model, and present UTA$^{GMS}$-GROUP and UTADIS$^{GMS}$-GROUP. These methods permit several DMs to cooperate in view of making a collective decision: UTA$^{GMS}$-GROUP — a choice and ranking decision, and UTADIS$^{GMS}$-GROUP — a sorting decision. For each DM who expresses her/his individual preference information we use the respective GMS method, and check whether the necessary and the possible relations or assignments hold for either at least one, or for all DMs. The collective results account for the preferences expressed by each DM. However, we avoid discussions of DMs on technical parameters, and rather consider two levels of certainty for the results. The first one is related to the consequences of preference information provided by each DM on the outcome. The other involves the subset of DMs agreeing for a specific outcome. Thus, we reason in terms of necessary and possible outcomes and coalitions of DMs, and we arrive at four types of results:

- *necessary–necessary*, i.e. result confirmed by all compatible instances of the preference model for all DMs;
- *necessary–possible*, i.e. result confirmed by all compatible instances of the preference model for at least one DM;
- *possible–necessary*, i.e. result confirmed by at least one compatible instance of the preference model for all DMs;
- *possible–possible*, i.e. result confirmed by at least one compatible instance of the preference model for at least one DM.

In this way, robust ordinal regression is used to investigate spaces of consensus and disagreement between DMs.

The paper is organized in the following way. In the next section, we recall the basic principles of robust ordinal regression methods in the framework of MAUT for a single DM, i.e. UTA$^{GMS}$ and UTADIS$^{GMS}$. Section 3 is devoted to the new extension of robust ordinal regression for multiple criteria group decision. Precisely, we adapt this principle to group choice, ranking, and sorting problems within MAUT. In the following section, we consider the case of incompatibility. Section 5 provides examples showing how the presented methodology can be applied in practical decision support. The last section contains conclusions and prospects future developments.

## 2. Reminder on robust ordinal regression in the framework of multi-attribute utility theory

We are considering decision problems in which a finite set of alternatives $A=\{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ is evaluated on a consistent family of criteria $G=\{g_1, \ldots, g_j, \ldots, g_m\}$. Let $G_j$ denote the value set (scale) of criterion $g_j$, $j\in J=\{1, \ldots, m\}$. Consequently, $G(A)=\prod_{j\in J}G_j$ represents the evaluation space. From a pragmatic point of view, it is reasonable to assume that $G_j\subseteq\mathbb{R}$, for $j=1, \ldots, m$. Moreover, without loss of generality, we assume that the greater $g_j(a)$, the better solution $a$ on criterion $g_j$, for all $j\in J$, $a\in A$. Finally, increasingly ordered different values of $G_j$ are denoted as: $x_j^1, x_j^2, \ldots, x_j^{n_j}$ with $x_j^k<x_j^{k+1}$, $k=1, 2, \ldots, n_j-1$, $n_j\leq n$.

Multi-Attribute Utility Theory (MAUT) provides a theoretical foundation for preference modeling using a value function, which aggregates evaluations of alternatives on multiple criteria. In this paper, in order to represent preferences of the DM, we use a model in the form of an additive value function $U(a)=\sum_{j=1}^{m}u_j(g_j(a))\in[0,1]$, where $u_j$ is the marginal monotone value function for criterion $g_j$, $u_j(x_j^1)=0$, for all $j\in J$, and $\sum_{j=1}^{m}u_j(x_j^{n_j})=1$.

In this section, we recall two robust ordinal regression methods within MAUT. One of them is intended to deal with ranking and choice problems, whereas the other is intended to support decision processes related to sorting problems.

### 2.1. UTA$^{GMS}$: robust ordinal regression for ranking and choice problems

In multiple criteria ranking and choice problems, alternatives from $A$ are compared one to any other and the results express relative judgments with the use of comparative notions. In the choice problem, the aim is to select a subset of the best alternatives, while in the ranking problem, alternatives are to be ranked from the best to the worst, according to the preferences of the DM. The idea of considering the whole set of compatible value functions to deal with ranking and choice problems was originally introduced in the UTA$^{GMS}$ method [6], and further generalized in GRIP [4].

The UTA$^{GMS}$ procedure consists of three steps. It starts with the preference elicitation process, leads through the statement of appropriate ordinal regression problems and results in calculation of binary relations on the set of all alternatives. In this subsection, we recall a general scheme of the method without going into details, which are presented in [6]:

I. Ask the DM (let us denote her/him by $d_r$) for preference information in form of pairwise comparisons of some reference alternatives $a,b\in A^R_{d_r}\subseteq A$. The DM can state that $a$ is at least as good as (weakly preferred to) $b$ ($a\succsim_{d_r}b$), or $a$ is indifferent to $b$ ($a\sim_{d_r}b$), or $a$ is strictly preferred to $b$ ($a\succ_{d_r}b$).
In GRIP, the DM may additionally provide preferences of two other types: either a partial preorder $\succsim^*_{d_r}$ on $A^R_{d_r}\times A^R_{d_r}$, such that for $a,b,c,d\in A^R_{d_r}$, $(a,b)\succsim^*_{d_r}(c,d)$ means $a$ is preferred to $b$ at least as much as $c$ is preferred to $d$ by $d_r$, or a partial preorder $\succsim^*_{j,d_r}$ on $A^A_{d_r}\times A^A_{d_r}$, such that for $a,b,c,d\in A^R_{d_r}$,$(a,b)\succsim^*_{j,d_r}(c,d)$ means $a$ is preferred to $b$ at least as much as $c$ is preferred to $d$ by $d_r$ on criterion $g_j$, $j\in J$.

II. Formulate the ordinal regression problem to verify that the set of compatible value functions $\mathcal{U}_{A^R,d_r}$ is not empty.

III. Compute the necessary $a\succsim^N_{d_r}b$ and the possible $a\succsim^P_{d_r}b$ weak preference relations for all $a,b\in A$. On the basis of the set of all compatible value functions $\mathcal{U}_{A^R,d_r}$, two binary relations on the set of all alternatives $A$ are defined:

   – *necessary* weak preference relation $\succsim^N_{d_r}$, in case $U(a)\geq U(b)$ for all value functions $U\in\mathcal{U}_{A^R,d_r}$ compatible with preference information provided by $d_r$,