Decision Support Systems 52 (2011) 52–60

Contents lists available at ScienceDirect

Decision Support Systems

journal homepage: www.elsevier.com/locate/dss

A framework for dynamic multiple-criteria decision making

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ARTICLE INFO

Article history:
Received 29 November 2010
Received in revised form 2 May 2011
Accepted 15 May 2011
Available online 18 May 2011

Keywords:
Dynamic decision making
Decision support systems
Multiple-criteria decision making

ABSTRACT

The classic multiple-criteria decision making (MCDM) model assumes that, when taking a decision, the decision maker has defined a fixed set of criteria and is presented with a clear picture of all available alternatives. The task then reduces to computing the score of each alternative, thus producing a ranking, and choosing the one that maximizes this value.
However, most real-world decisions take place in a dynamic environment, where the final decision is only taken at the end of some exploratory process. Exploration of the problem is often beneficial, in that it may unveil previously unconsidered alternatives or criteria, as well as render some of them unnecessary.
In this paper we introduce a flexible framework for dynamic MCDM, based on the classic model, that can be applied to any dynamic decision process and which is illustrated by means of a small helicopter landing example. In addition, we outline a number of possible applications in very diverse fields, to highlight its versatility.

1. Introduction

Most real-world decision problems are dynamic, in the sense that the final decision is taken only at the end of some exploratory process, during which both alternatives and criteria may vary, as the examples of Section 6.1 testify.

However, the classic multiple-criteria decision making (MCDM) model is unable to capture this dynamicity, since it assumes that, before proceeding with the ranking, the decision maker must have identified fixed sets of criteria and alternatives. While, in principle, this model could be used in a dynamic setting by considering subsequent decisions to be completely independent one from the other, doing so would constitute a gross oversimplification of the way humans think about the fine interlinking that exists among decisions in a dynamic environment, in which earlier evaluations affect later ones.

The framework we propose in this paper aims to address this problem by extending the classic MCDM model in a flexible way that enables its use in very diverse fields requiring some form of dynamic decision making.

The rest of this paper is organized as follows. In Section 2, we briefly review the classic MCDM model and present the general theory our framework is set in. Subsequently, in Section 3, we delve into the crucial issue of choosing an appropriate aggregation function for this model, and present some well-known examples from the recent literature. We then give, in Section 4, a general overview of related work, before going into the details of our proposed framework in Section 5. To better illustrate our proposal, we make use of a numerical example (Section 6) and present a number of possible applications (Section 6.1).

2. Classic MCDM model

The classic multiple-criteria decision making (MCDM) model [18,42] prescribes ways of evaluating, prioritizing and selecting the most favorable alternative from a set of available ones that are characterized by multiple, usually conflicting, levels of achievement for a set of attributes. The final decision is made by considering both inter-attribute and intra-attribute comparisons, possibly involving trade-off mechanisms.

Mathematically, a typical MCDM problem with m alternatives and n criteria is modeled by the matrix

$$\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_m \end{bmatrix},$$

where $x_{ij} \in [0, 1]$ represents the level of achievement of alternative a_i, $i = 1, \ldots, m$ with respect to criterion c_j, $j = 1, \ldots, n$, with 0 interpreted as "no satisfaction" and 1 corresponding to "complete satisfaction". It is also common to introduce a weight vector $\mathbf{w} \in [0, 1]^n$, $\sum_{j=1}^{n} w_j = 1$ whose generic component w_j, $j = 1, \ldots, n$ is the weight associated to criterion c_j representing its relative importance.

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doi:10.1016/j.dss.2011.05.003

Evaluation of alternatives is performed by means of an aggregation function $f: [0, 1]^n \to [0, 1]$, which maps vectors of criteria values x_i, $i = 1, \ldots, m$ to the [0, 1] interval and satisfies, for all $\mathbf{x}, \mathbf{y} \in [0, 1]^n$,

$$\begin{cases} f(\underbrace{0, 0, \ldots, 0}_{\text{n times}}) = 0 \\ f(\underbrace{1, 1, \ldots, 1}_{\text{n times}}) = 1 \end{cases} \quad \text{(preservation of bounds)},$$

$$\mathbf{x} \le \mathbf{y} \Rightarrow f(\mathbf{x}) \le f(\mathbf{y}) \qquad \text{(monotonicity)}.$$

The resulting value is considered a score indicating how preferable the associated alternative is, with the common understanding that 0 corresponds to “no preference” and 1 to “strongest preference”. Given these scores, alternatives may then be ordered, thus producing a ranking, and the best one might be selected.

It is clear that the aggregation function chosen for distilling criteria values into a single score plays a crucial role in this model, which in turn means that its mathematical properties need to be better categorized and understood. For this reason, in the following section we will present some of the more commonly used aggregation functions, highlighting interesting properties and providing pointers to existing literature for the interested reader.

3. Aggregation functions

As we have seen in the previous section, the key component of the classic MCDM model is the aggregation function used to associate a single score to each alternative by distilling the different evaluations (one for each criterion). It is thus easy to understand that the mathematical properties of this function will have a direct impact on the produced values and, therefore, on the final ranking of alternatives.

In the rest of this section we will present well-known aggregation functions, highlighting interesting properties such as full or partial reinforcement [42], which might prove useful in the decision process.

For more information on the broad field of aggregation functions, as well as for identifying a set of general guidelines to help select one, the interested reader should refer to [40,2,10,44,7,33,3]; our exposition will broadly follow [3].

3.1. Averaging aggregation functions

Averaging aggregation functions are probably the most commonly used aggregation functions. An aggregation function f is averaging if, for every $\mathbf{x}$, $\min(\mathbf{x}) \le f(\mathbf{x}) \le \max(\mathbf{x})$.

A wide and well-known class of averaging aggregation functions is that of means, which includes the arithmetic, quasi-arithmetic, geometric, harmonic and power means, as well as their weighted counterparts. Another family of averaging aggregation functions, introduced by Yager [39] and especially popular in the fuzzy sets community, is that of Ordered Weighted Averaging functions (OWA), which associate weights to values rather than particular inputs.

When criteria cannot be considered preferentially independent, as is often the case, a natural choice for the aggregation function is the discrete Choquet integral [15–17], which is able to model the importance of single criteria as well as of subsets of criteria. Underlying the Choquet integral is a monotonic set function, called capacity [9], that plays a role similar to that of a weight vector in traditional weighted arithmetic means.

Another interesting approach that should be mentioned is that of mixture operators [26,24], which extend weighted averaging operators by considering weighting functions defined on the aggregation domain instead of constant weights. Depending on the type of weighting function used, one can for example penalize poorly satisfied attributes, and reward well-satisfied ones. Two kinds of functions have been considered in this context, namely linear and quadratic weight generating functions [25,27].

Note, however, that these aggregation functions are in general not associative, and will thus not be the subject of further discussion in this review as they are not suited for the progressive aggregation process introduced later in this work.

3.2. Conjunctive aggregation functions

As their name implies, conjunctive aggregation functions are used to model conjunction, i.e. the logical *and*. They do not allow for compensation of low scores by other, higher scores, as it is the case, for example, of obtaining a driving license, for which one has to pass *both* the theory and the driving tests.

Therefore, their output is bound from above by the smallest input value, that is, for every $\mathbf{x}$, $f(\mathbf{x}) \le \min(\mathbf{x})$.

3.2.1. Triangular norms

The prototypical example of a conjunctive aggregation function is the so-called triangular norm, or t-norm. It was first introduced by Menger [20] as an operation for the fusion of distribution functions on statistical metric spaces, and its current definition, due to Schweizer and Sklar [32], requires associativity, symmetry and neutral element 1.

Four basic examples of t-norms are the minimum, the product, the Łukasiewicz t-norm and the drastic product [3]. The weakest and the strongest t-norms are the drastic product and the minimum, respectively; for every $\mathbf{x}$ and every t-norm T, it holds that $T_D(\mathbf{x}) \le T(\mathbf{x}) \le T_{\min}(\mathbf{x})$.

3.2.2. Parametric t-norms

Many families of related t-norms are defined by explicit formulas depending on some parameter. The main families of parametric t-norms are Hamacher's [44], Yager's [38] and Sugeno–Weber's [35], some of which include the basic t-norms as limiting cases.

3.3. Disjunctive aggregation functions

Disjunctive aggregation functions behave the opposite of conjunctive ones, in that satisfaction of *any* criteria is enough by itself, although positive inputs may reinforce one another. As their name implies, they are used to model disjunction, i.e. the logical *or*.

Therefore, their output is bound from below by the largest input value, that is, for every $\mathbf{x}$, $f(\mathbf{x}) \ge \max(\mathbf{x})$.

3.3.1. Triangular conorms

The dual aggregation function of a triangular norm is called a triangular conorm, or t-conorm. The current definition, again due to Schweizer and Sklar [32], requires associativity, symmetry and neutral element 0.

Four basic examples of t-conorms are the maximum, the probabilistic sum, the Łukasiewicz t-conorm and the drastic sum [3]. The weakest and the strongest t-conorms are the maximum and the drastic sum, respectively; for every $\mathbf{x}$ and every t-conorm S, it holds that $S_{\max}(\mathbf{x}) \le S(\mathbf{x}) \le S_D(\mathbf{x})$.

3.3.2. Parametric t-conorms

As for t-norms, many families of related t-conorms are defined by explicit formulas depending on some parameter. The main families of parametric t-conorms are again Hamacher's [44], Yager's [38] and Sugeno–Weber's [35], some of which include the basic t-conorms as limiting cases.