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Some extensions of the precise consistency consensus matrix

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ABSTRACT

The Precise Consensus Consistency Matrix (PCCM) is an AHP-Group Decision Making (AHP-GDM) tool, defined by Aguarón et al. [2] and developed in a local context (a single criterion) in which the decision makers are assigned the same weights. Using the Row Geometric Mean as the prioritisation procedure, consensus is sought between the different decision makers when the modifications of their initial positions or judgements are guaranteed to be within the range of values accepted for a given inconsistency level. This paper upgrades the algorithm initially proposed for obtaining the PCCM in two ways: (i) it considers the case of different weights for the decision makers; and (ii) it strengthens the idea of consistency in the design of the algorithm. One of the drawbacks of this decisional tool is that it is sometimes impossible to achieve a complete matrix. To address this, we propose a procedure for attaining a complete common consensus judgement matrix or, at least, a matrix with the minimum number of entries that are required to derive the priorities. Finally, we compare the results obtained when applying the extensions of the PCCM with those obtained using the two traditional procedures (AIJ and AIP) usually employed in AHP-GDM. In order to do this, we use a set of indicators that measure the violations in consistency of the group pairwise matrices and the compatibility between the individuals and group positions in four cases associated with two scenarios (weighted and non-weighted decision makers) and two situations (complete and incomplete PCCMs).

1. Introduction

The collaborative resolution of decisional problems (decision making involving multiple actors) and the evaluation of the associated intangible aspects (or the integration of tangible and intangible aspects in the formal models) are two of the most significant issues of the Knowledge Society [19].

A fundamental concept in decision making with multiple actors [2, 10,17,21,32], particularly with regard to Group Decision Making (GDM), is consensus. Consensus refers to the approach, model, tools, and procedures for deriving the final group priority vector [21]. If we understand consensus as agreement between the actors implicated in the resolution of the problem, agreement usually refers to collective preferences, although, on occasions, it can refer to the procedures followed for obtaining them, which are themselves based on individual preferences. If it is not possible to act as a homogenous group under the principle of consensus (a characteristic of group decision making), it is usual to seek collective preferences, so that the compatibility of individual preferences is as high as possible [25,33,34] and the modifications of the individual preferences that are required to fall in line with the group are minimised. This favours the overall acceptability of the collective result for the individuals implicated in the resolution of the problem.

These two criteria are normally followed in Negotiated Decision Making (NDM), under the principle of agreement [6,17]. Nevertheless, whilst it would be useful to resolve this semantic question as soon as possible, in the scientific literature on group decision making, the term consensus is commonly employed to reflect the idea of agreement or compatibility between individual and collective preferences [4,9,12,34].

Of the different multicriteria approaches followed for decision making, one that best captures [6] the two basic issues inherent in the Knowledge Society (multiple actors and the integration of intangible aspects) is the Analytic Hierarchy Process (AHP). The process allows the application of most of the different perspectives (determinist, stochastic, fuzzy etc.) used in the scientific literature with regard to the search for consensus [5–7,12,13,15–17,22,23,26,27,30,34,35].

AHP [23,24] is one of the most frequently employed approaches, both from theoretical and practical point of view. Two of its most important characteristics are: (i) it offers the possibility to evaluate the consistency of the judgements emitted in order to capture the preferences of the decision makers; and (ii) its correct behaviour in multiple actor decision making contexts. Defined as the cardinal transitivity of judgements [23], the concept of consistency differentiates AHP from other multicriteria techniques and it has been widely examined in the literature on AHP [8,17]. In multiactor decision making, AHP perfectly adapts to the three contexts that are contemplated [2,17,21]: Group Decision

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Making (GDM), Negotiated Decision Making (NDM), and Systematic Decision Making (SDM). For all three, a variety of procedures have been developed for evaluating compatibility between individual and collective positions.

A number of attempts have been made to integrate consistency and consensus (or compatibility) in multiple actor decision making using AHP [12,17,18,34]. With the aim of reaching collective positions close to the individuals' initial positions, that is to say, to preserve the initial positions as much as possible, Moreno-Jiménez et al. [17,18] proposed a new decisional tool for collective decision making – the Consistency Consensus Matrix (CCM) – that identifies the core consistency of the group decision. This interval judgement matrix does not have to be complete and it does not even have to connect the nodes. Aguarón et al. [2] further refined the tool, defining the Precise Consistency Consensus Matrix (PCCM) which is able to select a precise value for each judgement interval in such a way that the quantity of slack that remains free for successive algorithm iterations is the maximum possible. The PCCM was designed for a local context (a single criterion) and, due to its mathematical properties,[2] under the Row Geometric Mean (RGM) prioritisation procedure. As with the CCM, the PCCM is not guaranteed to be complete or connected, although it is more democratic than the CCM as it includes more entries or cells in which there is consensus in consistency (non-null cells).

The original PCCM proposal [2] allowed for multiactor decision making in which all the actors were given equal weighting. In this paper, we present a series of PCCM extensions that complement the original definition and permit the assignation of different weights to the decision makers in addition to strengthening the concept of consistency in the design of the algorithm. At the same time, we suggest a number of methods for connecting an unconnected matrix and to complete the PCCMs that have empty cells.

There are other models that use varying perspectives of consistency which allow the construction of a consensus matrix in AHP-GDM with multiplicative preference relations [12,34]. The main differences between these models and the new tool (PCCM) advanced for the AHP-GDM are that: (i) the PCCM seeks to maximise the number of entries of the consensus matrix that belongs to the consistency stability intervals of all the decision makers; and (ii) it guarantees that the consensus matrix entries are found within the levels permitted for fixed inconsistency in the process. This avoids the rejection (veto) inherent in moving away from the initial positions to a degree that is superior to that which is assumed by the accepted inconsistency level.

The remainder of this paper is structured as follows: Section 2 outlines the initial PCCM proposal [2]; Section 3 presents the extensions; Section 4 offers a series of numerical examples to illustrate the application of the extensions; and Section 5 details the most significant results and conclusions of the work.

2. The Precise Consensus Consistency Matrix (PCCM)

The two classic approaches followed in AHP-group decision making [13,17,22,23] are: (i) the Aggregation of Individual Judgements (AIJ) which constructs a pairwise comparison matrix for the group from which the priority vector is calculated by following any of the existing prioritisation procedures; and (ii) the Aggregation of Individual Priorities (AIP), in which the group priorities are obtained by aggregating the individual priorities (the Weighted Geometric Mean is most commonly used as the aggregation procedure).

A number of different measures have been put forward for evaluating the inconsistency of the decision makers when eliciting their judgements with AHP (it is not necessary for them to be perfectly consistent or transitive). Given a pairwise comparison matrix $A = (a_{ij})$, $i,j = 1,\ldots,n$, A is said to be consistent if there is cardinal transitivity between the judgements, that is to say, if $a_{ij} \cdot a_{jk} = a_{ik}\ \forall i,j,k$ [23]. Saaty suggests the Consistency Ratio for measuring the inconsistency if the Eigenvector method has been used to obtain the local priorities. When the local priorities have been derived by using the Row Geometric Mean (RGM), Aguarón and Moreno-Jiménez [3], basing themselves on Crawford and Williams [11], advocate the Geometric Consistency Index (GCI).

Given a pairwise comparison matrix $A = (a_{ij})$, $i,j = 1,\ldots,n$, and let $w = (w_j)$, $j = 1,\ldots,n$, be the corresponding priority vector obtained

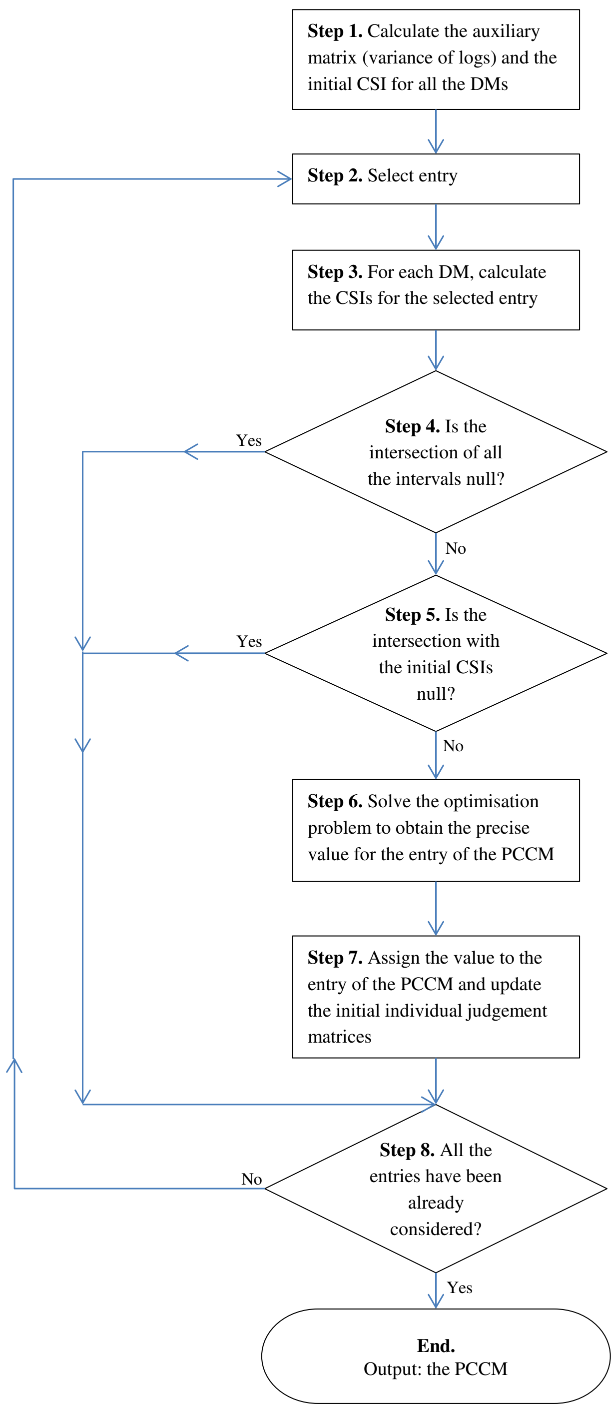

Fig. 1. The PCCM algorithm.

[2] The RGM's advantageous properties are most evident when calculating stability intervals for priority and consistency; in addition there is its relationship with traditional Eigenvector Method (EGVM), the fact that it requires less computational effort and, in group decisions, it provides the same results for the AIJ and AIP methods.