Decision Support Systems 53 (2012) 660–674

Contents lists available at SciVerse ScienceDirect

Decision Support Systems

journal homepage: www.elsevier.com/locate/dss

Multiple Criteria Hierarchy Process in Robust Ordinal Regression

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ARTICLE INFO

Article history:
Received 3 August 2011
Received in revised form 5 January 2012
Accepted 13 March 2012
Available online 28 March 2012

Keywords:
Multiple Criteria Decision Aiding
Hierarchy of criteria
Multiple Criteria Hierarchy Process
Robust Ordinal Regression
Preference modeling

ABSTRACT

A great majority of methods designed for Multiple Criteria Decision Aiding (MCDA) assume that all evaluation criteria are considered at the same level, however, it is often the case that a practical application is imposing a hierarchical structure of criteria. The hierarchy helps decomposing complex decision making problems into smaller and manageable subtasks, and thus, it is very attractive for users. To handle the hierarchy of criteria in MCDA, we propose a methodology called Multiple Criteria Hierarchy Process (MCHP) which permits consideration of preference relations with respect to a subset of criteria at any level of the hierarchy. MCHP can be applied to any MCDA method. In this paper, we apply MCHP to Robust Ordinal Regression (ROR) being a family of MCDA methods that takes into account all sets of parameters of an assumed preference model, which are compatible with preference information elicited by a Decision Maker (DM). As a result of ROR, one gets necessary and possible preference relations in the set of alternatives, which hold for all compatible sets of parameters or for at least one compatible set of parameters, respectively. Applying MCHP to ROR one gets to know not only necessary and possible preference relations with respect to the whole set of criteria, but also necessary and possible preference relations related to subsets of criteria at different levels of the hierarchy. We also show how MCHP can be extended to handle group decision and interactions among criteria.

1. Introduction

It is well known that the dominance relation established in the set of alternatives evaluated on multiple criteria is the only objective information that comes out from a formulation of a multiple criteria decision problem (including sorting, ranking and choice). While dominance relation permits to eliminate many irrelevant (i.e. dominated) alternatives, it does not compare completely all of them, resulting in a situation where many alternatives remain incomparable. This situation may be addressed by taking into account preferences of a Decision Maker (DM). Therefore, all Multiple Criteria Decision Aiding (MCDA) methods (for state-of-the-art surveys on MCDA see [7]) require some preference information elicited by a DM. Information provided by a DM is used within a MCDA process to build a preference model which is then applied on a non-dominated (Pareto-optimal) set of alternatives to arrive at a recommendation.

A great majority of methods designed for MCDA, assume that all evaluation criteria are considered at the same level, however, it is often the case that a practical application is imposing a hierarchical structure of criteria. For example, in economic ranking, alternatives may be evaluated on indicators which aggregate evaluations on several sub-indicators, and these sub-indicators may aggregate another set of sub-indicators, etc. In this case, the marginal value functions may refer to all levels of the hierarchy, representing values of particular scores of the alternatives on indicators, sub-indicators, sub-sub-indicators, etc. Considering hierarchical, instead of flat, structure of criteria, permits decomposition of a complex decision problem into smaller problems involving less criteria. To handle the hierarchy of criteria, we introduce in this paper a Multiple Criteria Hierarchy Process (MCHP). The basic idea of MCHP relies on consideration of preference relations at each node of the hierarchy tree of criteria. These preference relations concern both the phase of eliciting preference information, and the phase of analyzing a final recommendation by the DM. Let us consider a very simple and well known preference model, the linear value function, which assigns to each alternative $a\in A$ the value $U(a)=w_1g_1(a)+\ldots+w_ng_n(a), w_i\geq 0, i=1,\ldots n$, where $g_i(a)$ is an evaluation of alternative a on criterion $g_i, i=1,\ldots,n$. If in the phase of eliciting preference information, the DM declares that alternative a is preferred to alternative b with respect to a criterion which, in a node of the hierarchy tree, groups a set of sub-criteria $\mathcal{G}_{\mathbf{r}}$, this can be modeled as

$$\sum_{i\in\mathcal{G}_{\mathbf{r}}} w_i g_i(a) > \sum_{i\in\mathcal{G}_{\mathbf{r}}} w_i g_i(b),$$

which puts some constraints on the values of admissible weights w_i. In the phase of analyzing a final recommendation, even more important, MCHP shows preference relations $\succsim_{\mathbf{r}}$ on A with respect to the set of subcriteria $\mathcal{G}_{\mathbf{r}}$, such that, for all $a,b\in A$,

$$a\succsim_{\mathbf{r}} b \Leftrightarrow \sum_{i\in\mathcal{G}_{\mathbf{r}}} w_i g_i(a)\geq \sum_{i\in\mathcal{G}_{\mathbf{r}}} w_i g_i(b),$$

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doi:10.1016/j.dss.2012.03.004

where $a \succsim_{\mathbf{r}} b$ reads alternative a is at least as good as alternative b on the set of subcriteria $\mathcal{G}_{\mathbf{r}}$. Analyzing the preference relation $\succsim_{\mathbf{r}}$ is very useful in any decision aiding process because it permits to look into structural elements of the overall preference relation $\succsim$ taking into account the whole set of criteria, and justify better the final recommendation. For example, in a decision problem related to evaluation of students, one can say not only that student a is comprehensively preferred to student b, i.e. $a \succ b$ (where $\succ$ is the asymmetric part of $\succsim$; analogously, in the following, $\succ_{\mathbf{r}}$ is the asymmetric part of $\succsim_{\mathbf{r}}$), but also that a is comprehensively preferred to b because a is preferred to b on subsets of subjects (subcriteria) related to Mathematics and Physics, i.e. $a \succ_{Mathematics} b$ and $a \succ_{Physics} b$, even if b is preferred to a on subjects related to Humanities, i.e. $b \succ_{Humanities} a$. Moreover, one can also say that, for example, a is preferred to b on the subset of subjects related to Mathematics because, considering Analysis and Algebra as subjects (sub-criteria) related to Mathematics, a is preferred to b on Analysis, i.e. $a \succ_{Analysis} b$, and this is enough to compensate the fact that b is preferred to a on Algebra, i.e. $b \succ_{Algebra} a$. Since partial preference relations $\succsim_{Mathematics}$, $\succsim_{Physics}$, $\succsim_{Humanities}$, $\succsim_{Analysis}$, $\succsim_{Algebra}$, and so on, can be constructed using any MCDA methodology, this shows the universal character of MCHP.

In this paper, in order to show the useful features of MCHP, we apply this methodology to a recently proposed family of MCDA methods, called Robust Ordinal Regression (ROR) [8,9,11,12]. Basic ideas of ROR can be summarized as follows. To deal with a multiple criteria decision problem, Multiple Attribute Utility Theory (MAUT) [20] constructs a value function which assigns to each alternative a real number representing its degree of preferability. The first MCDA methods using the ordinal regression approach [4,25,28], aimed at finding one value function compatible with preference information provided by the DM (see, e.g., [6,18,22,24]). Most frequently additive value functions have been considered, i.e. functions obtained by summing up marginal value functions corresponding to particular criteria. For example, in [18], each marginal value function is a piecewise-linear one. Remark that in case of ordinal regression the preference information is always indirect.

In ordinal regression, and also in ROR, the preference information elicited by the DM is indirect, i.e. the DM provides decision examples, like preferential pairwise comparisons of some selected alternatives. This type of preference information is opposed to the direct one, which is composed of values of parameters of the assumed preference model, like weights or trade-off rates of the weighted sum model. Research indicates that indirect preference elicitation requires less cognitive effort from the DM than the direct one, and thus, it becomes more and more popular.

When building, via ordinal regression, a value function compatible with indirect preference information given as pairwise comparisons of some selected alternatives, one encounters a problem of plurality of compatible value functions. Until recently, the usual practice was to select only one of the compatible value functions, either by the DM or using some mathematical tools for finding a "central" value function. In general, however, each compatible value function gives a different ranking of the considered set of alternatives, and thus, it is reasonable to investigate what is the consequence of applying all compatible value functions on the whole set of considered alternatives. For this reason, ROR takes into account all compatible value functions simultaneously. In this context, two preference relations are considered:

– possible preference relation, for which alternative a is possibly preferred to alternative b if a is at least as good as b for at least one compatible value function, and
– necessary preference relation, for which alternative a is necessarily preferred to alternative b if a is at least as good as b for all compatible value functions.

The first method that applied the concept of ROR was UTA^{GMS} [9]: it takes into account pairwise comparisons of alternatives provided by a DM; GRIP [8] was its generalization taking into account not only pairwise comparisons, but also intensities of preference; ROR has been also applied to sorting problems [11], and it has been adapted to other preference models, like outranking relation [14,15] and non additive integrals [1].

Applying MCHP to ROR, permits to consider preference information at each level of the hierarchy in the phase of eliciting preference information. Moreover, putting together MCHP and ROR, permits to define necessary and possible preference relations at each node of the hierarchy tree. This gives an insight into evolution of the necessary and possible preference relations along the hierarchy tree. In fact, if we know that a is not necessarily comprehensively preferred to b, with MCHP we can find at which level a particular subcriterion opposes to the conclusion that a is necessarily preferred to b. All the properties that hold for the "flat" version of ROR methods are also valid in the hierarchical context, and other properties that are characteristic to the hierarchical context are given in this paper.

The paper is structured in this way: Section 2 describes some basic concepts of the MCHP; Section 3 describes the GRIP method adapted to the hierarchical context; in Section 4 we present the properties of necessary and possible preference relations; Section 5 describes the concept of intensity of preference and most representative value function; in Section 6 we present a didactic example; in Section 7 we present some extensions of the hierarchical ROR; Section 8 ends the paper with conclusions.

2. Multiple Criteria Hierarchy Process (MCHP)

In MCHP, we consider a set $\mathcal{G}$ of hierarchically ordered criteria, i.e. all criteria are not considered at the same level, but they are distributed over l different levels (see Fig. 1). At level 1, there are first level criteria called root criteria. Each root criterion has its own hierarchy tree. The leaves of each hierarchy tree are at the last level l and they are called elementary subcriteria. Thus, in graph theory terms, the whole hierarchy is a forest. We will use the following notation:

- $A = \{a, b, c, \ldots\}$ is the finite set of alternatives,
- l is the number of levels in the hierarchy of criteria,
- $\mathcal{G}$ is the set of all criteria at all considered levels,
- $\mathcal{I}_{\mathcal{G}}$ is the set of indices of particular criteria representing position of criteria in the hierarchy,
- m is the number of the first level criteria, $G_1, \ldots, G_m$,
- $G_{\mathbf{r}} \in \mathcal{G}$, with $\mathbf{r} = (i_1, \ldots, i_h) \in \mathcal{I}_{\mathcal{G}}$, denotes a subcriterion of the first level criterion G_{i_1} at level h; the first level criteria are denoted by G_{i_1}, $i_1 = 1, \ldots, m$,
- $n(\mathbf{r})$ is the number of subcriteria of $G_{\mathbf{r}}$ in the subsequent level, i.e. the direct subcriteria of $G_{\mathbf{r}}$ are $G_{(\mathbf{r},1)}, \ldots, G_{(\mathbf{r},n(\mathbf{r}))}$,
- $g_{\mathbf{t}}: A \to \mathbb{R}$, with $\mathbf{t} = (i_1, \ldots, i_l) \in \mathcal{I}_{\mathcal{G}}$, denotes an elementary subcriterion of the first level criterion G_{i_1}, i.e. a criterion at level l of the hierarchy tree of G_{i_1},
- EL is the set of indices of all elementary subcriteria:

$$EL = \{\mathbf{t} = (i_1, \ldots, i_l) \in \mathcal{I}_{\mathcal{G}}\} \text{ where } \begin{cases} i_1 = 1, \ldots, m \\ i_2 = 1, \ldots, n(i_1) \\ \ldots\ldots \\ i_l = 1, \ldots, n(i_1, \ldots, i_{l-1}) \end{cases}$$

- $E(G_{\mathbf{r}})$ is the set of indices of elementary subcriteria descending from $G_{\mathbf{r}}$, i.e.

$$E(G_{\mathbf{r}}) = \{(\mathbf{r}, i_{h+1}, \ldots, i_l) \in \mathcal{I}_{\mathcal{G}}\} \text{ where } \begin{cases} i_{h+1} = 1, \ldots, n(\mathbf{r}) \\ \ldots\ldots \\ i_l = 1, \ldots, n(\mathbf{r}, i_{h+1}, \ldots, i_{l-1}) \end{cases}$$

thus, $E(G_{\mathbf{r}}) \subseteq EL$,