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Fermi-Pasta-Ulam auto recurrence in the description of the electrical activity of the heart

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ABSTRACT

The authors proposed and mathematically described model of a new type of the Fermi-Pasta-Ulam recurrence (the FPU auto recurrence) and hypothesized an adequate description of the heart's electrical dynamics within the observed phenomenon. The dynamics of the FPU auto recurrence making appropriate electrical dynamics of the normal functioning of the heart in the form of an electrocardiogram (ECG) was obtained by a computer model study. The model solutions in the form of the FPU auto recurrence – ECG Fourier spectrum were evaluated for resistance to external disturbances in the form of random effects, as well as periodic perturbation at a frequency close to the heart beating rate of about 1 Hz. In addition, in order to simulate the dynamics of myocardial infarction model, studied the effect of the surface area of the myocardium on the stability and shape of the auto recurrence – ECG spectrum. It has been found that the intense external disturbing periodic impacts at a frequency of about 1 Hz lead to a sharp disturbance spectrum shape FPU auto recurrence – ECG structure. In addition, the decrease in the surface of the myocardium by 50% in the model led to the destruction of structures of the auto recurrence – ECG, which corresponds to the state of atrial myocardium. Research models have revealed a hypothetical basis of coronary heart disease in the form of increasing the energy of high-frequency harmonics spectrum of the auto recurrence by reducing the energy of low-frequency harmonic spectrum of the auto recurrence, which ultimately leads to a sharp decrease in myocardial contractility. In order to test the hypothesis has been studied more than 20,000 ECGs both healthy people and patients with cardiovascular disease. As a result of these studies, it was found that the dynamics of the electrical activity of normal functioning of the heart can be interpreted by the display of the detected by authors the FPU auto recurrence, and coronary heart disease is a violation of the energy ratio between the low and high frequency harmonics of the FPU auto recurrence Fourier spectrum equal to the ECG spectrum. Thus, the hypothesis has been confirmed.

Introduction

Canonical FPU recurrence was obtained as a result of numerical analysis of solutions of difference – differential equations describing the chain dynamics of nonlinearly coupled oscillators [1]. The model chain had no dissipation. To use this phenomenon in the description of the fundamental real dynamic processes, such as cardiac activity, the application required a model that can describe the refunds in autonomously functioning or self-oscillating systems, in which the dissipation is compensated by the system itself. A significant contribution to the study of the fundamental properties of the FPU recurrence was the work of American researchers Zabusky and Kruskal, who proposed to describe the FPU recurrence within the framework of the Korteweg-de Vries equation with periodically changing boundary and initial conditions in a closed chain [2]. The canonical FPU recurrence as well as the recurrence in the mathematical model of Zabusky and Kruskal is a return the picture of the spectrum of the system perturbations to its original form. However, the canonical FPU recurrence holds only for the first 5 modes [1], while in the real world dynamic systems, such as the heart, there is a great diversity of natural frequencies, among which are the groups of low and high frequencies. In this regard, the use of only low-frequency properties of the FPU recurrence for modeling the dynamics of heart activity is insufficient. In terms of expansion the properties of the FPU recurrence American researchers Lichtenberg and Lieberman in computer study of the dynamics of the discrete solutions of the sine – Gordon equation [3] the high frequency FPU recurrence has been found for the range of 20–35 modes, which made it possible to combine both types of the FPU recurrences into a body of the full FPU recurrence [4].

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Hypothesis and its mathematical model

By using the properties of the full FPU recurrence in ECG simulation should be noted that the functioning of the heart occurs in the self-oscillating mode, implying the existence of a similar principle of dynamics in the FPU recurrence structure used for simulation of cardiac activity. The first equation which describes self-excited oscillations, the Van der Pol's [5], and its two main solutions are the low-frequency (relaxation) and high frequency (harmonic) was derived. Using the two types of solutions of the Van der Pol equation, and the results of Zabusky and Kruskal and Lichtenberg and Lieberman works it is possible to simplify the solution of the KdV as cnoidal waves, replacing them with the solutions of the Van der Pol, close to harmonic, and low-frequency periodic solutions of the sine – Gordon – on relaxation solutions of the Van der Pol equation. Combining both equations in parametric and additive ways, we get the solution of equations involving Van der Pol as a simple, so-called FPU auto recurrence. Unlike canonical FPU recurrence, the auto recurrence periodically reflects the picture of the internal auto perturbation generated by nonlinear elements of a distributed self-oscillatory system, in particular by myocardial tissue cells electrical activity. External perturbations in the form of initial conditions affecting the system, in particular to the myocardium, are recorded in the dynamics of the system, in particular in the Fourier spectrum of the ECG, and periodically arise in its spectrum at the background of the main range of auto recurrence. Considering the above, and by applying the theorem on the possibility of replacing the wave links to delayed [6] in connection with a final time of propagation of electrical excitation along the myocardium, can be in the simplest approximation to present a mathematical model of the electrical activity of the heart (ECG) in the form of the FPU auto recurrence described under the related Van der Pol coupled equations with delay [4], having harmonic and relaxation solutions:

$$\frac{d^2M_1}{dt^2} - a_1(1-Y_1)\frac{dM_1}{dt} + \omega_1^2(1+\alpha_1 M_2)M_1 = c_1\frac{d^2M_2}{dt^2} + d_1F_1 + d_2F_2$$
$$b_1Y_1 + T_1\frac{dY_1}{dt} = M_1^2$$

$$\frac{d^2M_2}{dt^2} - a_2(1-Y_2)\frac{dM_2}{dt} + \omega_2^2(1+\alpha_2 M_1)M_2 = c_2\frac{d^2M_1}{dt^2} + d_1F_1 + d_2F_2$$
$$b_2Y_2 + T_2\frac{dY_2}{dt} = M_2^2$$

where M_1 – is the magnitude proportional to the dynamic electric potential of the whole myocardium, M_2 – is the magnitude proportional to the dynamic electric potential of the myocyte locality on the surface of the myocardium, Y_1 – is the magnitude proportional to the time lag which requires the propagation of the electric impulse in the myocardium, Y_2 – is the magnitude proportional to the time lag which requires the propagation of the electric impulse in the myocyte locality, b_1 – is the magnitude proportional to the square of the myocardium, b_2 – is the magnitude proportional to the square of the myocyte locality, T_1 – is the magnitude proportional to the time of the myocardium contraction, T_2 – is the magnitude proportional to the period of oscillations in the myocyte locality, ω_1, ω_2 – are the beating frequencies equal to 1, F_1 – is the perturbation function of the resonant external medium effect on the heart at the frequency of about 1 Hz, F_2 – is the random external function, $c_1, c_2, d_1, d_2, \alpha_1, \alpha_2$ – are the constants.

Computer study of the system (1) shows that in the intact myocardium and in the absence of external influences with a frequency of 1 Hz, the model reproduces auto recurrence, similar in shape to a healthy human electrocardiogram (Fig. 1). Fourier spectrum model auto recurrence – ECG is shown in Fig. 2.

Fig. 3 shows the Fourier spectrum of the real electrocardiogram of a healthy person.

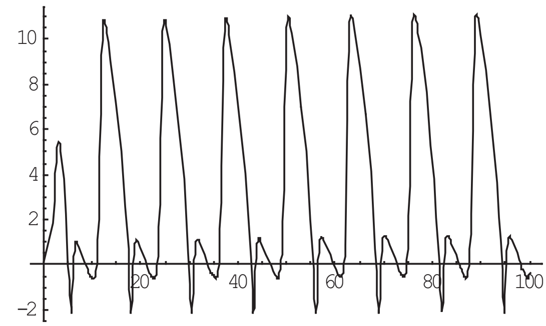

Fig. 1. Model auto recurrence – normal ECG. Horiz.axis – frequency vert.axis – amplitude, conditional units.

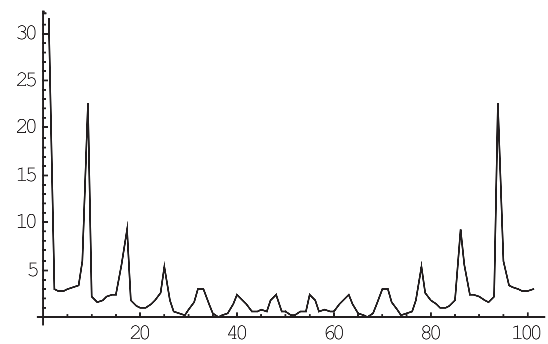

Fig. 2. Fourier spectrum model auto recurrence – normal ECG without any external influence, and the intact myocardium. Horiz.axis – frequency vert.axis – amplitude, conditional units.

As shown by mathematical modeling, the electrical activity of the heart represents the first formulated by the authors the FPU auto recurrence bearing individual character, and containing in its structure a spectral picture of the heart electrical dynamics corresponding to its physiological state and its pathological processes, patterns which periodically appear in the Fourier spectra of the auto recurrence.The spectrum shown in Fig. 3, in qualitative agreement with the model range of the FPU auto recurrence equal to normal ECG, the resulting research model described by the system (1). It should be emphasized, that the auto recurrence Fourier spectra of the ECG of healthy individuals contain a low-frequency portion of the range of 1–5 Hz, corresponding to the canonical FPU recurrence [1] and the high-frequency portion of the range of 20–35 Hz, corresponding to the so-called high-frequency FPU recurrence, other words ECG is a biological example of a full [6] or the FPU auto recurrence. It should be noted high stability of the auto recurrence to external disturbances of random nature. Thus, increasing the amplitude of the random nature of the external exposure (F_2) order of magnitude (factor of 0.1 to 1) causes only minor changes in the ECG spectrum.

Auto recurrence (Figs. 4 and 5).As a result of external influence is a random process only an insignificant occurrence of quasi-periodic change in the spectrum of harmonics phase Auto Recurrence as shown in the Figs. 4 and 5.At the same time, increasing the amplitude of the external effects on the heart rate of approximately 1 Hz to 50% (0.1–0.5) leads to the destruction of the overall structure of the ECG auto recurrence to the level of the heart functioning, called atrial fibrillation (Figs. 6 and 7). To close the picture results in a decrease in the surface model infarction by 50% (b_1). Described state of the model can be called a myocardial infarction.

As you can see from the picture of the real atrial fibrillation (Fig. 8), it is characterized by the complete destruction of the pic-