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Identifying degrees of freedom in pushbroom bundle adjustment

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ABSTRACT

An analytical method of detecting and enumerating degrees of freedom (DOF) in bundle adjustment (BA) is developed, and used to analyze several scene geometries for pushbroom cameras. While it is well recognized in the photogrammetric community that BA can be poorly constrained, especially with pushbroom imagery, the literature is absent of techniques to study the DOF in BA except through numerical analysis, which does not delineate them except through the suggestion of their existence in degraded results. The analytical methods presented here provide insight into the DOF. The method is based on finding the singular values, and the associated singular directions, of a Hessian matrix from the minimization process in BA. These directions are scrutinized as possible DOF along which the minimal cost of BA will not change from the true solution. To demonstrate the use of the methods six different scene geometries are analyzed, some of which are commonly employed by current remote sensing satellites. Several of the scene geometries are shown to have multiple DOF beyond the well known scaling and absolute position and orientation that are not recoverable from stereo imagery without exterior orientation control of the cameras or ground control points. As a simple example, the effects of these DOF images from the HiRISE camera on the Mars Reconnaissance Orbiter are used along with the associated ephemeris data to demonstrate possible distortions in the results of a terrain reconstruction problem.

1. Introduction

Bundle adjustment (BA) is commonly used with stereo imagery in terrestrial and exploration applications to refine the estimates of camera exterior orientation (EO). The process is often used to improve products such as georeferenced imagery and digital elevation models (DEMs) produced through triangulation of corresponding pixels in stereo pairs. It is a well known process formulated in the late 1950s by Brown and Schmid (Brown, 1958; Schmid, 1958) that has been extensively used in the photogrammetric community since then (Mikhail et al., 2001; McGlone et al., 2004). More recently it has also been documented in several texts and extended papers due to its increasing popularity within the computer vision community (Hartley and Zisserman, 2004; Triggs et al., 2000). BA for pushbroom imagery is more complicated than that for frame cameras, but is increasingly being used to refine EO parameters within the exploration community that are then fed into in semi-automated terrain reconstruction algorithms (Kirk et al., 2007; Li et al., 2008), which may result in poor accuracy without indication. However, the adjustments made by BA for pushbroom imagery may have degrees of freedom (DOF) in the solution space beyond those commonly known from frame camera EO refinement. These DOF allow convergence to solutions that are inconsistent with the true camera EO. They depend on the geometric relationship between the cameras' EO used to obtain the imagery. Furthermore, the residuals minimized in BA do not provide any indication that an incorrect solution has been found. While several studies, for example, Hofmann et al. (1984) have investigated the practical constraints of BA with numerical techniques, there is no analytical discussion of this problem in the open literature. This work provides an analytical technique for studying this problem and uses it to analyze several geometries.

For orbital imagery of Earth, ground control points (GCPs) are commonly used to help constrain the BA process. An example is the use of GCP in the production of the recently released global DEM produced from ASTER data (Hirano et al., 2003; ASTER GDEM Validation Team, 2009) for Earth. However, the use of GCPs incurs significant expense. First precise position measurements need to be collected, either from GPS or existing maps. Then a human operator must manually identify the GCPs in the images. These requirements not only increase cost but also prevent full automation of terrain modeling. In addition, for certain terrains,

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GCPs are difficult to obtain e.g. arctic ice sheets (Korona et al., 2009) or extraterrestrial terrains (Kirk et al., 2007).

To reduce manual processing time, ephemeris data (ED) generated from navigation system measurements on the spacecraft are increasingly being relied upon more heavily than GCPs. An example is SPOT5 satellite (Gleyzes et al., 2003). For many systems, navigational accuracy has been greatly increased by the Global Positioning System (GPS). However, in exploration, such as for Mars (Li et al., 2008) or the Moon (Chin et al., 2007; Haruyama et al., 2008, 2009) this is not true, and GCPs are difficult to obtain and not accurate. Recent publications on DEM accuracy (Bouillon et al., 2006; Korona et al., 2009) for SPOT5, which only makes use of GCPs periodically over calibration sites, report good accuracy. SPOT5 cameras were designed to be forward and aft looking to produce stereo imagery for terrain reconstruction, and similar camera configurations are being used for exploration orbiters (Haruyama et al., 2008, 2009). However, it is dangerous to assume that the multi-line camera configuration of SPOT5 is responsible for its accuracy in DEM generation, and that this translates to the exploration orbiters. We will show that BA for the three line imaging geometry without either GCPs or ED has a degree of freedom not explicitly identified in other literature. Therefore, the inaccurate GCPs and ED of in exploration missions may result in poor adjustment of the EO parameters.

A few authors make mention of DOF in BA, for example Triggs et al. briefly discuss the need to reduce the number of "gauge freedoms" (Triggs et al., 2000) and several others refer to the commonly known scale and absolute position and orientation DOF without control points. A few other authors have performed simulation studies to determine the accuracy of terrain reconstruction for specific instruments under different conditions (Ebner et al., 1999). However, the literature is largely absent of any analytical discussion of the other DOF in pushbroom BA. This paper provides a method of finding such DOF with the intention of opening the door toward understanding their effects in a fundamental way rather than based on simulations with a large set of parameters to vary.

1.1. Bundle adjustment

This work is concerned with BA for pushbroom cameras whereby EO parameters are refined using two images of the same terrain captured from different vantage points. Pioneering work was performed in this area by Hofmann et al. (1984) and Ebner and Strunz (1988). Recently this topic was covered by Triggs et al. (2000) and Hartley and Zisserman (2004).

Using camera EO (position and attitude) DEMs, orthorectified images, and other similar products can be produced through projections of the image rays. Like all measurements, camera EO measurements are corrupted by noise. Therefore, to improve the quality of these products, the camera EO estimates are often adjusted to enhance scene consistency using BA. BA uses only small set of corresponding features from overlapping images called "tie points" to improve the scene consistency. This section details the structure of the BA algorithm to set the framework for discussion of the pitfalls of scene geometry in applying BA to stereo pushbroom imagery.

The typical formulation of BA adjusts all scene parameters including the camera positions, $\mathbf{p}_c$, camera attitudes, $\boldsymbol{\phi}_c = [\omega\,\phi\,\kappa]^T$, and feature (tie points) locations in 3D, $\mathbf{p}_f$, to minimize a scalar cost function. The cost function is typically a sum of squared errors. Individual error terms for tie points are measured in the camera's focal plane as the difference between measured location of the feature in the image, $\mathbf{p}_m$, and that predicted by the current state of the estimated parameters, $\mathbf{p}_b$. Prediction of a feature's image location is accomplished through back-projection of $\mathbf{p}_f$, into the camera's focal plane. Without lens distortion, the pinhole camera model describes how 3D points are imaged. With this model, the projection of $\mathbf{p}_f$ into the camera focal plane is given by

$$\mathbf{p}_b = \begin{bmatrix} u_b \\ v_b \end{bmatrix} = -\frac{f}{d_z}\begin{bmatrix} d_x \\ d_y \end{bmatrix} \tag{1}$$

where f is the camera focal length and

$$\mathbf{d} = [d_x\, d_y\, d_z]^T = \mathbf{R}_{cr}(\mathbf{p}_f - \mathbf{p}_c) \tag{2}$$

and

$$\mathbf{R}_{cr} = \begin{bmatrix} c_\phi c_\kappa & c_\omega s_\kappa + s_\omega s_\phi c_\kappa & s_\omega s_\kappa - c_\omega s_\phi c_\kappa \\ -c_\phi s_\kappa & c_\omega c_\kappa - s_\omega s_\phi s_\kappa & s_\omega c_\kappa + c_\omega s_\phi s_\kappa \\ s_\phi & -s_\omega c_\phi & c_\omega c_\phi \end{bmatrix}. \tag{3}$$

Here $c_* = \cos(*)$ and $s_* = \sin(*)$. The back-projection error between an observed feature, $\mathbf{p}_m$, and its prediction, $\mathbf{p}_b$, is given by

$$\mathbf{e} = \mathbf{p}_m - \mathbf{p}_b = \begin{bmatrix} u_m \\ v_m \end{bmatrix} - \begin{bmatrix} u_b \\ v_b \end{bmatrix} = \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}. \tag{4}$$

The scalar cost function measuring the scene consistency is the sum of squared back-projection errors,

$$C = \mathbf{c}^T\mathbf{c}, \tag{5}$$

where, for m images with n tie points, the cost vector is

$$\mathbf{c} = [\mathbf{e}_1^T \,\ldots\, \mathbf{e}_m^T]^T \tag{6}$$

where

$$\mathbf{e}_i = [\mathbf{e}_{1,i}^T \,\ldots\, \mathbf{e}_{n,i}^T]^T \quad \text{for } i = 1 \ldots m. \tag{7}$$

Here, for simplicity, it is assumed that all n tie points are observed in all m cameras. For n tie points and m cameras, if we define the vector of parameters to be estimated as

$$\mathbf{k} = [\mathbf{p}_{f_1}^T \,\ldots\, \mathbf{p}_{f_n}^T\, \mathbf{p}_{c_1}^T\, \boldsymbol{\phi}_{c_1}^T \,\ldots\, \mathbf{p}_{c_m}^T\, \boldsymbol{\phi}_{c_m}^T]^T, \tag{8}$$

then plugging Eq. (1) into Eq. (4) and the result into Eq. (6) provides a scalar cost as a function of the scene parameters, $C = C(\mathbf{k})$.

Unlike frame cameras which capture an entire image from a single EO, pushbroom cameras sweep out images by traveling through space. Each image line is acquired with a slightly different camera EO. Therefore, EO parameters associated with each image are a function of image line number or time, t. In this paper we express each camera EO parameter as a nominal function of time plus a time dependent offset. The offset is modeled as a second order polynomial in t whose coefficients are adjusted by BA. The camera EO is given by,

$$\begin{bmatrix} \mathbf{p}_c(t) \\ \boldsymbol{\phi}_c(t) \end{bmatrix} = \begin{bmatrix} x_0(t) + \alpha_x t^2 + \beta_x t + \gamma_x \\ y_0(t) + \alpha_y t^2 + \beta_y t + \gamma_y \\ z_0(t) + \alpha_z t^2 + \beta_z t + \gamma_z \\ \omega_0(t) + \alpha_\omega t^2 + \beta_\omega t + \gamma_\omega \\ \phi_0(t) + \alpha_\phi t^2 + \beta_\phi t + \gamma_\phi \\ \kappa_0(t) + \alpha_\kappa t^2 + \beta_\kappa t + \gamma_\kappa \end{bmatrix} \tag{9}$$

where the nominal trajectories $x_0(t)$, $y_0(t)$, $z_0(t)$, $\omega_0(t)$, $\phi_0(t)$ and $\kappa_0(t)$ are assumed to be known measurements provided by a navigation system. The parameter vector is then expanded to include the polynomial coefficients,

$$\mathbf{k} = [\mathbf{p}_{f_1}^T \,\ldots\, \mathbf{p}_{f_n}^T\, [\alpha_{x_1} \,\ldots\, \gamma_{\kappa_1}] \,\ldots\, [\alpha_{x_m} \,\ldots\, \gamma_{\kappa_m}]]^T. \tag{10}$$

The necessary addition of these path parameters significantly decreases the constraints on the scene geometry as compared to BA for frame cameras. We will show that this permits the