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Stable predictive controller design with pole location specification

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Abstract

The paper presents a stable predictive controller design based on solving a linear matrix inequality. The main idea is the augmentation of the state-space model with model prediction to obtain predicted values of output and input signals. Performance specification is a desired closed-loop poles location in a circle in the complex plane. Structure of the practical implementation is provided. Simulation example of light intensity control proves applicability of the controller design.

Keywords: Predictive control; Linear matrix Inequality; Pole location

1. Introduction

Predictive control (MPC) is one of the most popular advanced controller design techniques widely used in industrial applications. There exists a lot of modifications of the basic idea of using predicted system input and output (state) signals in the control algorithm. One of the first was Model predictive heuristic control presented by Richalet et al. (1978), Dynamic matrix control (Prett et al., 1982) and generalized predictive control (GPC) developed by Clarke et al. (1987, 1987). Extensive overview of MPC algorithms can be found in Maciejowski (2002), Rossiter (2004), Camacho and Bordons (1999).

Standard MPC is an optimization based algorithm which solves minimization of a cost function subject to constraints in every sampling period. Such a controller requires big on-line computational load and does not guarantee the closed-loop stability. Currently there is several MPC algorithms which can guarantee stability or even robust stability. Excellent survey can be found in Bemporad and Morari (1999), Mayne et al. (2000).

This paper shows the basic predictive controller structure and design based on the ideas of Veselý et al. (2010), Nguyen et al. (2011). The augmented system model with prediction model and integral term provides a systematic way for MPC design. The created structure has a form of a standard state-space model, incorporates set-point tracking

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with zero steady state error and allows to use any of the well know state or output feedback design method. Although, it does not contains in its basic form any constraints handling it creates a good keystone for additional extensions.

The paper is organized as follows. In Section 2, the main problem and system augmentation is presented. Section 3 explains the feedback gain calculation. In Section 4, the practical implementation of the controller algorithm is explained and finally an example of light intensity control is in Section 5.

For simplicity several notational conventions will be adopted: I denotes the identity matrix with a corresponding dimensions, $M>0$ ($M<0$) denotes positive (negative) definiteness of matrix M and the standard expression $x(k+h|k)$ is shortened to $x(k+h)$ which denotes h steps ahead prediction of x calculated in the time step k.

2. Problem formulation

At first an augmented system model is created. It provides set-point tracking and output prediction with arbitrary prediction horizon. The standard system structure is preserved that allows one to use all standard techniques known for the state-space controller design.

Consider the system model is

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $y(k) \in R^l$ are state, control input and output variables of the system respectively.

To achieve set-point tracking an integrator $q(k+1) = q(k) + w(k) - y(k)$ is added to the system model and the augmented system is:

$$\begin{aligned} x_a(k+1) &= A_a x_a(k) + B_a u(k) + B_w w(k) \\ y_a(k) &= C_a x_a(k) \end{aligned} \tag{2}$$

where

$$\begin{aligned} x_a(k) &= \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}, \quad y_a(k) = \begin{bmatrix} y(k) \\ q(k) \end{bmatrix} \\ A_a &= \begin{bmatrix} A & 0 \\ -C & I \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix} \\ C_a &= \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ I \end{bmatrix} \end{aligned} \tag{3}$$

For the prediction $x(k+2)$ one obtains:

$$\begin{aligned} x_a(k+2) &= A_a x_a(k+1) + B_a u(k+1) + B_w w(k+1) = A_a^2 x_a(k) + A_a B_a u(k) + Bu(k+1) + A_a B_w w(k) + B_w w(k+1) \\ y_a(k+1) &= C_a x_a(k+1) \end{aligned}$$

Similarly the prediction for time $k+h$ is

$$\begin{aligned} x_a(k+1+h) &= A_a^{h+1} x_a(k) + \sum_{i=0}^{h} A_a^{h-i} B_a u(k+i) + \sum_{i=0}^{h} A_a^{h-i} B_w w(k+i) \\ y_a(k+h) &= C_a x_a(k+h) \end{aligned} \tag{4}$$

Let the prediction horizon be N then from (4) system with model prediction is:

$$x_f(k+1) = A_f x_f(k) + B_f u_f(k) + B_{wf} w_f(k) \tag{5}$$

$$y_f(k) = C_f x_f(k) \tag{6}$$