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Closed-form solutions for estimating a rigid motion from plane correspondences extracted from point clouds

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ABSTRACT

Registration is often a prerequisite step in processing point clouds. While planar surfaces are suitable features for registration, most of the existing plane-based registration methods rely on iterative solutions for the estimation of transformation parameters from plane correspondences. This paper presents a new closed-form solution for the estimation of a rigid motion from a set of point–plane correspondences. The role of normalization is investigated and its importance for accurate plane fitting and plane-based registration is shown. The paper also presents a thorough evaluation of the closed-form solutions and compares their performance with the iterative solution in terms of accuracy, robustness, stability and efficiency. The results suggest that the closed-form solution based on point–plane correspondences should be the method of choice in point cloud registration as it is significantly faster than the iterative solution, and performs as well as or better than the iterative solution in most situations. The normalization of the point coordinates is also recommended as an essential preprocessing step for point cloud registration. An implementation of the closed-form solutions in MATLAB is available at: http://people.eng.unimelb.edu.au/kkhoshelham/research.html#directmotion.

1. Introduction

Mapping natural and built environments usually involves acquiring dense and detailed spatial data in the form of point clouds. To capture all facets of the objects of interest often multiple point clouds are needed. Hence, a prerequisite step in processing point clouds is to align two or more recordings and transform them into one common coordinate system. This process is called registration. While many approaches to registration exist, a typical approach in practice is to select (manually or automatically) a number of corresponding points from the two point clouds, and estimate a transformation between the point clouds by minimizing the Euclidean distance between the corresponding points.

Using point correspondences usually works best for roughly-aligned dense point clouds or when the correspondences are measured by markers. When the point density is low and the points are noisy, using point correspondences leads to an inaccurate registration. In such cases, using plane correspondences will result in a more accurate registration (Rusinkiewicz and Levoy, 2001). Plane fitting methods are less influenced by noise and low density of the points, and can be made robust to possible outliers. Plane-based registration is particularly suitable for point clouds of man-made objects, since such objects typically have polyhedral shapes and planar surfaces.

Plane-based registration of point clouds has been a topic of growing interest in recent years (Grant et al., 2012; Habib et al., 2010; Khoshelham, 2010; Lichti and Chow, 2013; Pathak et al., 2010; Van der Sande et al., 2010; Taguchi et al., 2012). It consists of three main steps: extraction of planes, establishing correspondence, and estimating a transformation using the corresponding plane pairs. The focus of this paper is the estimation step. Most of the existing approaches use iterative solutions for the estimation of transformation parameters from plane correspondences. Iterative solutions require an initial approximate estimate of the transformation parameters, and the iterations might converge slowly, converge to a local optimum or not converge at all. Also, since the estimation is often a necessary step for finding or updating the correspondences, an iterative solution will increase the computational cost of the whole registration process. The efficiency is particularly important when registering data acquired by range cameras and depth sensors, such as the Kinect, which

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can capture point clouds at video frame rates (Khoshelham et al., 2013).

In contrast, closed-form solutions are independent of the initial approximation, and are more efficient as they don't involve iteration. While several closed-form solutions based on point correspondences exist and have been evaluated (Eggert et al., 1997), a thorough study of closed-form solutions based on plane correspondences is not available. This paper has three main contributions:

- a new closed-form solution based on point–plane correspondences is proposed;
- the role of normalization in accurate plane fitting and estimation from planes is investigated;
- a thorough evaluation of the closed-form solutions is presented, including comparisons with the iterative solution in terms of accuracy, robustness, stability and efficiency.

The paper proceeds with a review of related research on point cloud registration in Section 2. The iterative and closed-form methods for transformation estimation based on plane correspondences are described in Section 3. Experimental evaluation of the methods is presented in Section 4. The paper concludes with a discussion in Section 5.

2. Related work

Early work on estimating a transformation from point correspondences goes back to studies of the absolute orientation problem in photogrammetry by Thompson (1958), and later by Schut (1960), Sansò (1973) and Horn (1987), who used different quaternion representations to derive linear equations for the estimation of rotation parameters. Estimating a rotation matrix from point observations has also been referred to as Procrustes problem (Hurley and Cattell, 1962) and Wahba's problem (Wahba, 1965), for which solutions based on Singular Value Decomposition (SVD) have been proposed (Markley, 1988; Schönemann, 1966). Arun et al. (1987) developed a closed-form method using SVD to estimate a rigid motion from point correspondences. Horn et al. (1988) proposed a closed-form solution for the special case where point correspondences are co-planar. Eggert et al. (1997) provide a comparative evaluation of four closed-form solutions that use point correspondences. The popular Iterative Closest Point (ICP) algorithm (Besl and McKay, 1992) makes use of Horn's solution that is based on unit quaternions (Horn, 1987).

Chen and Medioni (1992) developed a different ICP algorithm, in which the transformation is estimated by minimizing the sum of squared orthogonal distances from the points in the source surface to the corresponding tangent planes in the destination surface. This point–plane metric has been shown to perform significantly better than the point-point metric in terms of accuracy and convergence rate (Rusinkiewicz and Levoy, 2001). However, unlike the point-point metric, the estimation based on the point–plane metric has been done in an iterative fashion, making the algorithm computationally more expensive. In the past decade, many works on point cloud registration have shown a preference for the point–plane metric (Grant et al., 2012; Gruen and Akca, 2005; Habib et al., 2010; Lichti and Chow, 2013; Mitra et al., 2004; Rabbani et al., 2007), though all these methods use iterative solutions. Olsson et al. (2006) developed a method for minimizing the point–plane metric, which gives a direct estimate of translation, but is iterative in estimating rotation. Van der Sande et al. (2010) proposed a closed-form solution for minimizing point–plane distances between overlapping point clouds acquired by an airborne Lidar mapping system. Their solution gives a 3D affine transformation rather than a rigid motion, with the undesired consequence that it can deform the point cloud. A more recent development is the set of minimal solutions of Ramalingam and Taguchi (2013) for estimating a rigid motion from several configurations of point–plane correspondences. These minimal solutions handle specific configurations of point–plane correspondences differently, and as such are not applicable to arbitrary numbers of points and planes.

Another approach to plane-based registration is by using plane–plane correspondences. In this approach, the error metric for minimization is the difference between the corresponding parameters of the corresponding planes. There are closed-form solutions for minimizing the plane–plane metric, which take advantage of the closed-form solutions for point correspondences. In these solutions, the normal vectors of the corresponding planes are treated as 3D points, enabling the application of point-based closed-form solutions. Gregor and Whitaker (2001) use the SVD approach of Arun et al. (1987) to estimate the rotation matrix from corresponding plane normals, whereas Brenner et al. (2008) and Pathak et al. (2010) use the quaternion method of Horn (1987). Although the closed-form plane–plane solutions have been shown to result in a satisfactory registration, they have not been thoroughly evaluated in terms of robustness to noise and improper plane configurations.

In summary, the literature suggests that a thorough study of closed-form solutions for estimating a rigid motion from plane correspondences is not available. In this paper, we propose a new closed-form solution based on singular value decomposition for estimating a rigid motion from point–plane correspondences, and evaluate the performance of the closed-form solutions in comparison to the iterative solution.

3. Estimation of a rigid motion from plane correspondences

3.1. Preliminaries

This section provides mathematical preliminaries on the transformation of points and planes in 3D space, the Kronecker product and the singular value decomposition. In the following mathematical expressions, matrices are denoted by bold uppercase letters, column vectors are denoted by bold lowercase letters and scalars are written in italics. Points and planes in 3D space are represented as column vectors.

The transformation between two point clouds typically consists of a 3D rotation **R** and a 3D translation **t**. It is convenient to combine these two in a transformation matrix **H** of homogeneous coordinates:

$$\tilde{\mathbf{p}}^d = \mathbf{H}\,\tilde{\mathbf{p}}^s = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \tilde{\mathbf{p}}^s \tag{1}$$

where the superscripts s and d denote respectively the source and destination coordinate system, and $\tilde{\mathbf{p}} = (x, y, z, 1)^T$ is the homogeneous representation of a point in 3D space. The transformation **H** is usually referred to as a rigid motion. For a set of points that lie on a plane, the plane equation can be written as:

$$\boldsymbol{\pi}^{\mathrm{T}}\tilde{\mathbf{p}} = 0 \tag{2}$$

where $\boldsymbol{\pi} = (\mathbf{n}^{\mathrm{T}}, -\rho)^{\mathrm{T}}$ is the homogeneous representation of the plane defined by a normal vector $\mathbf{n} = (n_x, n_y, n_z)^{\mathrm{T}}$ of unit length and its perpendicular distance ρ from the origin. From (1) and (2), it can be seen that if **H** transforms a set of co-planar points from a source coordinate system s to a destination coordinate system d, the corresponding transformation for the planes is (Khoshelham and Gorte, 2009):

$$\boldsymbol{\pi}^d = \mathbf{H}^{-\mathrm{T}}\boldsymbol{\pi}^s \tag{3}$$

where $\mathbf{H}^{-\mathrm{T}}$ denotes the inverse transpose of **H**.