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Algebraic reasoning for the enhancement of data-driven building reconstructions

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ABSTRACT

Data-driven approaches for the reconstruction of buildings feature the flexibility needed to capture objects of arbitrary shape. To recognize man-made structures, geometric relations such as orthogonality or parallelism have to be detected. These constraints are typically formulated as sets of multivariate polynomials. For the enforcement of the constraints within an adjustment process, a set of independent and consistent geometric constraints has to be determined. Gröbner bases are an ideal tool to identify such sets exactly. A complete workflow for geometric reasoning is presented to obtain boundary representations of solids based on given point clouds. The constraints are formulated in homogeneous coordinates, which results in simple polynomials suitable for the successful derivation of Gröbner bases for algebraic reasoning. Strategies for the reduction of the algebraical complexity are presented. To enforce the constraints, an adjustment model is introduced, which is able to cope with homogeneous coordinates along with their singular covariance matrices. The feasibility and the potential of the approach are demonstrated by the analysis of a real data set.

1. Introduction

1.1. Motivation

Virtual city models are the base for many applications with geometric questions such as flooding calculations, occlusion analysis or the simulation of wave propagation. The model instances are usually derived from uncertain point observations. Since these point clouds contain no explicit representation of structure, reverse engineering has to be applied to obtain as-built model instances of man-made objects.

In general, reconstruction approaches are roughly categorized as either model- or data-driven: "Model-driven approaches choose configurable building blocks from a library of pre-defined templates, determine their roof shape parameters to best fit the given data, and possibly combine them with other blocks to generate more complex shapes. Pure data-driven approaches aggregate the measured points to form higher-order primitives (usually planar regions) and combine them to form surface models without any shape restrictions" (Kada and Wichmann, 2013).

Data-driven approaches—such as the direct construction of polyhedrons—feature the flexibility needed to capture objects of arbitrary shape, unknown in advance. Unfortunately, the corresponding procedures can be very complicated. Often they produce non-planar faces and unaesthetic substructures (Haala and Kada, 2010).

The detection of regular structures and the enforcement of corresponding geometric constraints yields a "beautification", convenient for many visualization tasks. But the fact that established model instances must feature topological consistency, completeness, and integrity is more important (Mäntylä, 1987).

The constraints must be consistent and independent for consideration within an adjustment process. Gröbner bases are the perfect tool to determine such sets. The challenge is to keep the algebraic complexity low to obtain tolerable computing time and memory consumption.

1.2. Related work

For a survey of the major approaches for Geometric Constraint Solving, i.e., graph-based, logic-based, and algebraic methods, please refer to Hoffmann and Joan-Arinyo (2005). General descriptions of the concept of Gröbner bases are from a mathematical point of view (Eisenbud, 1995) and from a more application-

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oriented point of view (Heck, 1996), whereas the construction of the Gröbner bases leads back to the original work of Buchberger et al. (1988).

The formulation of constraints as multivariate polynomials can be found in Brenner (2005), for instance, where Gröbner bases are used to analyze the interactive modification of constrained 2D objects. In 3D, constraints for entities in homogeneous representation along with their covariance matrices are provided by Heuel (2004). An adjustment model to solve constrained problems formulated in homogeneous coordinates efficiently can be found in Meidow (2014).

In the context of 3D building modeling, Rottensteiner et al. (2005) use manually introduced constraints for the delineation of planar roof patches within an adjustment. Pohl et al. (2013), use a greedy algorithm to select a set of independent and consistent constraints automatically. Decisions are made based on estimated ranks and condition numbers, which leads to numerical problems and inexact results for large-scale problems.

In Loch-Dehbi and Plümer (2011) and Loch-Dehbi and Plümer (2009), independent constraints are found by automatic theorem proving using Wu's method (Wu, 1986). The feasibility is demonstrated, but no real data sets have been analyzed.

1.3. Contribution

We present a complete work flow for reasoning based on uncertain 3D point observations. This includes feature extraction by grouping of points, the detection of geometric constraints for man-made structures by hypotheses tests, the determination of independent and consistent constraints by the utilization of Gröbner bases, and the efficient enforcement of constraints found in this manner. Boundary representations of solids are derived by elaborated interaction of statistics, algebra, and numerical optimization.

To formulate geometric constraints, we exploit the calculus of projective geometry. The representation of geometric entities in homogeneous coordinates leads to simple constraints with often trivial Jacobians since the constraints are often bi-linear in the parameters. For the relations, we generate and test hypotheses, which avoids hardly interpretable thresholds.

The main contribution is the utilization of Gröbner bases to exactly determine independent and consistent constraints. We discuss strategies to reduce algebraic complexity by rephrasing and reasonably sequentially arranging the constraints.

To enforce the constraints, we introduce an adjustment model which is able to cope with homogeneous coordinates along with their singular covariance matrices. The size of the resulting normal equation system depends on the number of constraints only. Therefore, this optimization task can be solved efficiently.

The feasibility and usefulness of the approach, as well as some limitations, are demonstrated by the evaluation of real data sets. In doing so, we apply Singular, a computer algebra system for polynomial computations (Decker et al., 2012), which has been integrated in our framework.

1.4. Outline and notation

1.4.1. Outline

This paper is organized as follows: After motivating the use of projective geometry for our purposes in Section 2, we cover the complete work flow for geometric reasoning. Hence, we restrict our presentation to the consideration of planes. After describing the grouping of 3D points and the estimation of corresponding plane parameters in Section 3, we introduce geometric constraints and present the hypotheses generation and verification in Section 4. Section 5 treats the determination of sets of independent and consistent constraints by computer algebra, exploiting the concept of Gröbner bases. The results are then the starting point for the overall adjustment of planes, which enforces the set of identified constraints (Section 6). Section 7 presents results of experiments conducted using real data sets, and we close with a summary and the conclusions in Section 8.

1.4.2. Notation

We distinguish between the name of a geometric entity denoted by a calligraphic letter, e.g. $\mathcal{X}$ for a point, and its representation. To represent geometric entities, we use homogeneous vectors denoted by upright boldface letters, e.g., $\mathbf{a}$. Euclidean vectors and matrices are denoted by slanted boldface letters, e.g., $\boldsymbol{x}$ or $\boldsymbol{R}$. With homogeneous coordinates, "=" means an assignment or an equivalence up to a scaling factor $\lambda \neq 0$. With the skew-symmetric matrix $\boldsymbol{S}(\boldsymbol{x})$ we express the cross product $\boldsymbol{S}(\boldsymbol{x})\boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$ of two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. The dot product is denoted by $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x}^{\mathsf{T}}\boldsymbol{y}$.

2. Representations and normalizations

Geometric constraint can easily be expressed by multivariate polynomials. The algebraic complexity of these polynomials heavily depends on the specific representation of the involved geometric entities. For algebraic reasoning and for subsequent numerical optimization, we need both Euclidean and homogeneous representations. Since we are dealing with over-parametrizations, additional constraints or normalizations are needed to make these entities unique.

2.1. Representations

A 3D point in Euclidean representation is a coordinate triplet $\boldsymbol{x} = [x, y, z]^{\mathsf{T}} \in \mathbb{R}^3$. The homogeneous coordinates of a 3D point are then elements of the 4-vector

$$\mathbf{x} = \begin{bmatrix} \lambda \boldsymbol{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \lambda x \\ \lambda y \\ \lambda z \\ \lambda \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ t \end{bmatrix} \tag{1}$$

subject to the constraint $\|\mathbf{x}\|^2 = u^2 + v^2 + w^2 + t^2 \neq 0$ with $\lambda \neq 0$.

A convenient representation of a plane is its normal vector $\boldsymbol{n}$ with $\|\boldsymbol{n}\| = 1$ and the signed distance d of the plane to the origin of the coordinate system. The corresponding homogeneous vector reads

$$\mathbf{a} = \begin{bmatrix} \boldsymbol{n} \\ -d \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_h \\ a_0 \end{bmatrix} \tag{2}$$

with its homogeneous part $\boldsymbol{a}_h$ and its Euclidean part a_0.

The point-plane incidence

$$\langle \mathbf{x}, \mathbf{a} \rangle = \mathbf{x}^{\mathsf{T}}\mathbf{a} = 0 \tag{3}$$

is the base of the estimation of plane parameter values $\mathbf{a}$ for a given set of points. Furthermore, the formulation of the concurrence constraint for planes (explained below) is based on the dot product (3).

2.2. Spherical normalization

Given a 4-vector $\mathbf{a}$ representing a plane and its corresponding 4×4 covariance matrix $\boldsymbol{\Sigma}_{\mathrm{aa}}$, we obtain the spherical normalized vector

$$\bar{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|} \tag{4}$$

and its covariance matrix