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Adjustability and error propagation for true replacement sensor models

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Abstract

True replacement sensor models, or TRSM, are those based on construction of dense object–image grids using the rigorous physical sensor model. Photogrammetric exploitation of image sensing applies the TRSMs since, like the physical models, they possess the same three important characteristics: (1) Accurate ground-to-image function; (2) Rigorous error propagation that is essentially of the same accuracy as the physical model; and, (3) Adjustability, or the ability to upgrade the TRSM parameters when additional control information becomes available after replacing the physical model. The ground-to-image functions are commonly achieved via fitting rational polynomial coefficients, RPC, to the dense grids which encompass the entire ground volume covered by the image under consideration. A novel approach for rigorous error propagation, without using added parameters, has been developed at Purdue University. The approach resolves the problem of rank deficiency of the covariance matrix associated with RPC by theeigen-values and eigen-vectors approach. This paper summarizes the new approach and presents further development to address the adjustability characteristic. Results from its application to imagery from an aerial frame camera, an airborne pushbroom sensor, and a spaceborne linear array sensor, are presented for both simulated as well as real image data. The results show essentially negligible differences when compared to those from the rigorous physical sensor models.

Keywords: Sensor modeling; Replacement sensor models; Error propagation; Adjustability

1. Introduction

Image sensing remotely acquired from aircraft and spacecraft is a primary data acquisition for spatial information. Associated with the image is a physical/geometrical model which provides the mathematics for photogrammetric analysis including: 1) computing earth-related coordinates of the real objects from image measurements, and vice-versa, 2) determining stochastic characteristics of the computed coordinates, which is referred to as error propagation, and 3) upgrading the *a priori* available model parameters using additional control information, which we call model adjustability. The collinearity equation is the most commonly used form as the fundamental relationship for different types of passive sensors, including frame cameras (areal sensors), pushbroom sensors (linear sensors), and whiskbroom sensors (point sensors).

On the other hand, there exist sensor models that are purely mathematical. These sensor models are

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implemented on the basis of the physical/geometric sensor models. The latter models are used to construct dense grids that encompass the entire ground volume covered by the image under consideration and used to derive these mathematical models. Forms of those models and their exploitation are relatively simple compared to those of the original geometrical models which are often more complicated. The commonlyused forms are polynomial and rational polynomial functions of different degrees. Alternatively, the dense grid of ground–image coordinate pairs calculated using a geometrical model can be used with interpolators. To compute ground coordinates from imagemeasurements, and vice versa, the mathematical functions or grid/interpolator provide sufficiently close value to those from the original models. These purely mathematical models are known as true replacement sensor models, or TRSM, when they have the same three features of the original models: (1) Highly accurate fit to the grid; (2) Rigorous error propagation that is essentially of the same accuracy as the geometrical model; and (3) Adjustability, or the ability to upgrade the model parameters when additional control information becomes available after the replacement model has already been derived and the original geometrical model is no longer available.

1.1. Commonly used ground-to-image functions

The most commonly used function for TRSM is the rational polynomial function. Its form is as follows:

$$l = \frac{\sum_{k=0}^{l_{nz}} \sum_{j=0}^{l_{ny}} \sum_{i=0}^{l_{nx}} a_{ijk} X^i Y^j Z^k}{\sum_{k=0}^{l_{dz}} \sum_{j=0}^{l_{dy}} \sum_{i=0}^{l_{dx}} b_{ijk} X^i Y^j Z^k} \tag{1a}$$

$$s = \frac{\sum_{k=0}^{s_{nz}} \sum_{j=0}^{s_{ny}} \sum_{i=0}^{s_{nx}} c_{ijk} X^i Y^j Z^k}{\sum_{k=0}^{s_{dz}} \sum_{j=0}^{s_{dy}} \sum_{i=0}^{s_{dx}} d_{ijk} X^i Y^j Z^k} \tag{1b}$$

As a ground-to-image function, the input on the right hand side of the function contains rational polynomial coefficients (RPC), a, b, c, d, and normalized ground coordinates, X, Y, Z, and the function calculates normalized image coordinates, l, s (for line and sample). It is suggested in Förstner et al. (2004) that the maximum power for ground coordinates be 5, i.e., $0 \leq u_{nx}, u_{ny}, u_{nz}, u_{dx}, u_{dy}, u_{dz} \leq 5$. The coefficients are defined relative to normalized coordinates ofboth the ground and image points. Normalization for a coordinate, x, is defined as:

$$x_{\text{normalized}} = \frac{x - \text{offset}_x}{\text{scale factor}_x} \tag{2}$$

Although, rational polynomial functions can extend to any power, there are only a few forms commonly used in practice. One of them is the 78-coefficient rational polynomial function and another one is the direct linear transformation, which is an 11-coefficient special form of the 78-coefficient having only linear terms and a denominator that is common for line and sample coordinates. A matter of concern when using rational polynomial functions is the possibility of zero-crossings occurring in the denominator and, therefore, it must be checked for during the estimation of coefficients.

When fit accuracy allows, a numerator only polynomial can be specified to avoid denominator zero-crossing problems. Its form is exactly Eqs. (1a) and (1b) without the denominators, as follows:

$$l = \sum_{k=0}^{l_{nz}} \sum_{j=0}^{l_{ny}} \sum_{i=0}^{l_{nx}} a_{ijk} X^i Y^j Z^k \tag{3a}$$

$$s = \sum_{k=0}^{s_{nz}} \sum_{j=0}^{s_{ny}} \sum_{i=0}^{s_{nx}} b_{ijk} X^i Y^j Z^k \tag{3b}$$

Frequently, the highest power is not restricted to three as in the 78-coefficient case.

1.1.1. Coefficient estimation

To encapsulate the ground-to-image functionality of an original physical sensor model, TRSM coefficients are estimated by utilizing the physical sensor model's image support data. Using these data, a grid of ground point-image point correspondences is generated as the first step in creating the replacement sensor model. While the grid of image points extends through the entire image, the grid of ground points should extend both to cover the image footprint and a range in the vertical direction that coversthe expected terrain elevation range. After the generation, points in the grid are divided into two sets, fit points and check points. Fig. 1 depicts such a grid. The number of fit planes in each direction as shown in the figure may be different in the coefficient estimation of different images in order to obtain a required fit accuracy. For example, in Theiss et al. (2004), 9×9×9 grid was shown to provide perfect fit for an aerial frame photograph.

With a chosen replacement model such as one from those described in the previous section, coordinates of the ground point-image point correspondences in the fit grid are used to form the equations, that contain the coefficients as unknown. Because of the large number of points, the number of equations is generally much larger than the number of coefficients to be estimated and the unknowns are practically solved by using least squares estimation. We may use one of the three methods for solving for the coefficients as described in Förstner et al.