ISPRS Journal of Photogrammetry and Remote Sensing 111 (2016) 1–12

Contents lists available at ScienceDirect

ISPRS Journal of Photogrammetry and Remote Sensing

journal homepage: www.elsevier.com/locate/isprsjprs

# Surface-based matching of 3D point clouds with variable coordinates in source and target system

Xuming Ge *, Thomas Wunderlich

*Chair of Geodesy, Technische Universität München, Arcis-str. 21, D-80333, München, Germany*

ARTICLE INFO

*Article history:*
Received 10 June 2015
Received in revised form 31 October 2015
Accepted 1 November 2015
Available online 19 November 2015

*Keywords:*
3D surface matching
Surface registration
Point cloud
Laser scanning

ABSTRACT

The automatic co-registration of point clouds, representing three-dimensional (3D) surfaces, is an important technique in 3D reconstruction and is widely applied in many different disciplines. An alternative approach is proposed here that estimates the transformation parameters of one or more 3D search surfaces with respect to a 3D template surface. The approach uses the nonlinear Gauss–Helmert model, minimizing the quadratically constrained least squares problem. This approach has the ability to match arbitrarily oriented 3D surfaces captured from a number of different sensors, on different time-scales and at different resolutions. In addition to the 3D surface-matching paths, the mathematical model allows the precision of the point clouds to be assessed after adjustment. The error behavior of surfaces can also be investigated based on the proposed approach. Some practical examples are presented and the results are compared with the iterative closest point and the linear least-squares approaches to demonstrate the performance and benefits of the proposed technique.


## 1. Introduction

Surface registration is an intermediate, but crucial, step in the three-dimensional (3D) reconstruction of real objects. The terrestrial laser scanning technique is now widely applied in surveying engineering, photogrammetry, and other disciplines as its performance capabilities are advancing rapidly (Ebeling et al., 2011; Gordon and Lichti, 2007). Hundreds of different laser scanners with a great variety of measurement systems are now available; in the current fast-paced laser scanning market, these are updated almost every year. Even products of the same brand may have totally different specifications between different series, e.g. different sensors, different resolutions, different scales and different degrees of precision. An alternative approach to surface registration is therefore required with the ability to handle these different types of sensors, at different resolutions and at different degrees of precision. It is also necessary to analyze the error behavior of surfaces and to assess the registered observations after adjustment.

A cloud of point samples from the surface of an object is typically obtained from two or more points of view in different reference frames. Registration consists of the alignment of the search point set with the template point set by estimating the transformations between the datasets. The registration strategy may differ depending on whether the targets are used to provide the reference points in two clouds of point sets. In terms of whether initial information is required, registration techniques can be classified as either coarse or fine registration. In coarse registration, the main goal is to compute an initial estimate of the rigid motion between two corresponding clouds of 3D points; in fine registration, the goal is to obtain the most accurate solution possible. In fine registration, a higher-quality initial estimate is always required before the calculation. The scope of this paper will be limited to non-target fine registration.

Non-target fine registration is achieved by using a sufficient overlap of the point clouds in different datasets and minimizing the sum of the squares of the distance between the temporarily corresponding points in each iteration. A well-known approach to solving the problem is the iterative closest point (ICP) method (Besl and McKay, 1992; Zhang, 1994; Chen and Medioni, 1991, 1992). The implementation of the ICP method is based on the point-to-point or point-to-plane searching techniques and an estimation of the rigid transformation that aligns the pairs of nearest points in the two datasets. Although the ICP method is a powerful algorithm for non-target registration, it has obvious shortcomings e.g. low time efficiency and easily fall into a local minimum (Fusiello et al., 2002; Gruen and Acka, 2005; Salvi et al., 2007). Several variations and improvements have been introduced to the original version of the ICP concept to improve the algorithm in terms of the transformation accuracy, the convergence properties and the computational cost (Masuda and Yokoya, 1995;

* Corresponding author.

http://dx.doi.org/10.1016/j.isprsjprs.2015.11.001

Trucco et al., 1999; Greenspan and Godin, 2001; Sharp et al., 2002; Zinsser et al., 2003; Low, 2004; Grant et al., 2012).

Another powerful approach used to complete 3D surface matching originates from the least-squares matching (LSM) technique (Gruen, 1984, 1985a; Ackermann, 1984; Pertl, 1984). Surface patch matching in photogrammetry was first resolved by Gruen (1985a) using this technique. Multiple patch matching with two-dimensional (2D) images using the LSM technique has also been demonstrated by Gruen (1985b). Gruen and Acka (2005) reported a least-squares 3D (LS3D) surface matching approach. This approach was designed for arbitrary 3D surface data and is an extension of 2D least-squares image matching. Akca (2010) enhanced the LS3D approach in terms of the computational cost. However, as Grant et al. (2012) pointed out, the stochastic properties of the normal to the local surface are neglected in the LS3D approach. Gruen and Acka (2005) and Akca (2010) used the LS3D approach based on the generalized Gauss–Markov model to estimate the transformation parameters with the assumptions that the measurement errors have a simple stochastic character without bias and that only the components of the *target* surface are affected by these errors. The stochastic quantities of the *source* surface were neglected; the effect of these is minor if the *target* and *source* surfaces are generated by the same sensor or method with the same measurement error pattern. However, the principal hypothesis of a particular model matrix in the Gauss–Markov model is not necessarily satisfied in all applications.

We present here a new approach, an extension of the LS3D approach, to match two arbitrary 3D surfaces. Our proposed approach estimates the rigid-body transformation parameters between two corresponding point clouds using the nonlinear Gauss–Helmert (GH) model. We therefore called the proposed approach the GH-LS3D approach. In this GH-LS3D approach, the nonlinear GH model is introduced to address the so-called weighted total least-squares problem. The iteratively linearized GH model proposed by Pope (1972) was used to adjust error-in-variables (EIV) model problems in arbitrary 3D surface matching problems. By solving the least-squares problem within the GH model, we obtained the solution to the underlying nonlinear problem with a reasonable approximation and some of the potential pitfalls in the iterative adjustment of nonlinear problems were avoided (Pope, 1972). This provided an opportunity to analyze the error behavior of both surfaces and to assess the co-registered surfaces after adjustment.

Section 2 briefly defines the problems of registration and presents the mathematical formulation of the GH-LS3D approach, as well as describing the computational implementation of the proposed approach. Section 3 presents some experimental results based on the non-target fine registration of the point clouds of a terrestrial laser scanner to demonstrate the capabilities of the proposed approach. Some conclusions and further extensions are given in Section 4.

## 2. GH-LS3D surface matching

### 2.1. Statement of the problem and definition

This work concentrated on the pairwise alignment of two meshes, although multi-view registration (Pulli et al., 1997; Pulli, 1999) should also benefit from the approach described here. Let *target* and *source* refer to two partially overlapping scans (surfaces) in two different local coordinate frames. The task of registration is to estimate the transformation parameters that, when applied to the *target* points, best align the *source* and the *target*. Alignment is measured by an error function – for example, the minimization of the sum of the squares of the Euclidean distance. To measure the Euclidean distance, we need to select the correspondence between the *source* and the *target*. The ICP algorithm and its variants provide different schemes for choosing the corresponding points between the *source* and the *target*; they then use the correspondences to calculate the transformation parameters based on a rigid-body transformation. Thus the registration algorithms mainly consist of two steps: (1) matching and selection; and (2) computation of the transformation parameters. In the rest of this paper, we label these two steps *M* and *C*, respectively.

To start the registration algorithm, the program runs the *M*-step to update the *target* by approximations of the transformation parameters and selects the new version of the correspondences from the *source*. Then, in the *C*-step, the program estimates the updated transformation parameters using the correspondences from the last *M*-step. If the estimated transformation parameters do not change significantly in the *C*-step, or if the results reach a termination condition, the iteration will be terminated; otherwise the iteration returns to the *M*-step and uses the last updated transformation parameters instead of the previous values. The schemes to find ideal correspondences in the *M*-step can be applied in parallel in the ICP series and the LS3D approach. In the *T*-step in the ICP method, the goal function that minimizes the Euclidean distance by least squares is obtained indirectly by estimating and applying rigid transformations. In contrast, the LS3D approach formulates the goal function directly in a generalized Gauss–Markov model (Gruen and Acka, 2005). The GH-LS3D approach is an extension of the LS3D approach; the main improvement in the proposed approach is in terms of the *T*-step.

### 2.2. Mathematical model

The *source* and *target* refer to two partially overlapping scans that are digitized point by point in the two different local coordinate frames of the same object. Let $f(x,y,z)$ and $g(x,y,z)$ denote overlapping regions of the object in the *source* surface and the *target* surface, respectively. The problem of 3D surface matching based on the LSM statement is estimating the transformation parameters to align the target surface $g(x,y,z)$ with the source surface $f(x,y,z)$. If a matching is established between $f(x,y,z)$ and $g(x,y,z)$ then following equation holds:

$$f(x,y,z) = g(x,y,z) \tag{1}$$

According to Eq. (1), the *source* surface elements have corresponding elements in the *target* surface and vice versa. If we then assume that $e(x,y,z)$ is a true error vector between the two surfaces, then we can derive:

$$f(x,y,z) + e(x,y,z) = g(x,y,z) \tag{2}$$

Eq. (2) is the observation equation for LS3D. In GH-LS3D, the error vector $e(x,y,z)$ is divided into $e_f(x,y,z)$ and $e_g(x,y,z)$, which represent the error stemming from the *source* surface and *target* surface, respectively. Thus the observation equation for GH-LS3D is:

$$f(x,y,z) + e_f(x,y,z) = g\{(x,y,z) + e_g(x,y,z)\} \tag{3}$$

Eq. (3) is the condition equation with the measurement errors in all observations. Then the matching problem is to solve the following least squares problem:

$$\sum \left( ||e_f||^2 + ||g\{e_g\}||^2 \right) = \min \tag{4}$$

It is known that, in the Gauss–Markov model, only the components of the observation vector are affected by the measurement errors and these are calculated by the least-squares technique with a certain design matrix in a normal equation. A standard EIV model is a Gauss–Markov model with an uncertain design matrix in a