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New rigorous and flexible Fourier self-calibration models for airborne camera calibration

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ABSTRACT

This paper presents a new family of rigorous and flexible mathematical self-calibration additional parameters (APs) for airborne camera calibration. Photogrammetric self-calibration can – to a very large extent – be considered as a *function approximation* problem in mathematics. It is shown, that algebraic polynomials are less desirable for designing self-calibration APs due to the highly correlated terms. Based on the mathematical approximation theory, we suggest that Fourier series be the optimal mathematical basis functions for self-calibration purpose. A whole family of so-called *Fourier self-calibration APs* is developed, whose solid theoretical foundations are the Laplace's Equation and the Fourier Theorem. Fourier APs are orthogonal, rigorous, flexible, generic and efficient for calibrating the image distortion of frame-format airborne cameras. The high performance of Fourier APs is demonstrated in many practical tests on different camera systems, including the DMC, DMCII, UltracamX, UltracamXp and DigiCAM cameras. We illustrate the theoretical and practical advantages of the Fourier APs over the physical APs and the popular polynomial APs. The joint applications with physical models are promoted for specific applications as well. On account of the theoretical justifications and high practical performance, Fourier APs should be preferred for in situ airborne camera calibration.

1. Introduction

Camera calibration is an essential subject in photogrammetry and computer vision. Self-calibration by using additional parameters (APs) has been widely accepted and substantially utilized as an efficient calibration technique in photogrammetric community since the 1970s. Self-calibration plays a significant role in the automatic interior/exterior orientation of camera systems. Many studies were investigated on self-calibration APs in the last decades. Traditionally, two types of self-calibration APs were developed for *analogue* single-head camera calibration: physical and mathematical. The development of physical APs was mainly attributed to Brown (1971) for close-range camera calibration and these APs were later extended for aerial application (Brown, 1976). El-Hakim and Faig (1977) proposed mathematical APs by using spherical harmonics. Ebner (1976) and Grün (1978) built the algebraic polynomial APs of second and fourth orders, respectively. These early works showed the remarkable significance of self-calibration APs in reducing image residuals and improving accuracy, while the concerns were raised as well on overparameterization and high correlations (Ackermann, 1981; Kilpelä, 1981; Clarke and Fryer, 1998). The quite popular polynomial APs are often criticized as "have no foundations based on observable physical phenomena" (Clarke and Fryer, 1998; McGlone et al., 2004).

These APs, though being widely used for many years and even in digital era, might be however inadequate to fit the distinctive features of *digital* airborne cameras, such as push-broom, multi-head, virtual images composition and various image formats (Honkavaara et al., 2006; Cramer, 2009). A considerable progress was made for digital camera calibration. The self-calibration model was investigated in Fraser (1997) for close-range digital camera calibration. The self-calibration of large frame-format airborne cameras was investigated in Jacobsen (2007). Cramer (2009, 2010) reported comprehensive empirical tests, in which lots of different APs were employed to compensate image distortion (Jacobsen et al., 2010). Yet, inconveniences still remain. Firstly, many APs are purely the combinations of the traditional APs and might be inadequate. Secondly, some APs are designed and increasingly used without evident physical or mathematical foundations (Clarke and Fryer, 1998). Thirdly, some APs suffer for high correlations with the interior orientation (IO) parameters or other correction parameters (Kresse et al., 2006; Cramer, 2009). Last but not least, some are tailored for specific cameras concerning the manufacturing technologies. They can hardly be used to calibrate different cameras in a general sense.

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On the other hand, the successful incorporation of navigation sensors into camera system demands the overall system calibration, rather than calibrating image distortion only. The systematic effects, such as shift/drift and misalignments caused by direct georeferencing, must be calibrated for photogrammetric applications (Honkavaara, 2004; Cramer et al., 2010). A major challenge of the whole system calibration is decoupling different calibration effects. Serious considerations are desired on high correlations.

All the above motivate this work to progress airborne camera self-calibration. We point out, that photogrammetric self-calibration can – to a very large extent – be considered as a *function approximation* problem in mathematics. The distortion can thus be modelled by a linear combination of specific basis functions. An intrinsic deficiency of the polynomial self-calibration APs is revealed. Algebraic polynomials are not proper for the self-calibration purpose from the mathematical viewpoint. By examining different mathematical bases, Fourier series are favoured as the optimal mathematical basis functions for building self-calibration APs. Then, a whole family of so-called *Fourier self-calibration APs* is developed. Fourier APs are orthogonal, mathematically rigorous, flexible, generic and efficient for calibrating the image distortion of all frame airborne cameras. The performance of Fourier APs is evaluated in several empirical tests. The advantages of the Fourier APs are demonstrated over the physical APs and the polynomial APs.

The paper is organized as follows. The mathematical principle for building self-calibration APs is introduced and the Fourier APs are constructed in Section 2. Practical test results are demonstrated in Section 3, whereas comparisons are made in Section 4 between Fourier APs with other counterparts. Finally, the conclusions summarize the work.

2. Fourier self-calibration APs

The collinearity equations, which are the mathematical fundamental of photogrammetry, are given in

$$x = x_0 - c\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} + \Delta x + \varepsilon$$
$$y = y_0 - c\frac{r_{12}(X-X_0)+r_{22}(Y-Y_0)+r_{32}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} + \Delta y + \varepsilon \quad (1)$$

where Δx and Δy denote image distortion. The other notations can be seen in textbooks such like Kraus (2007). The distortion terms are two-variable functions, whose forms are unknown. They need to be approximated by parametric models, i.e., self-calibration APs.

The subsequent key work is to develop a proper parametric model which can accurately represent the actual distortion. As known, there are several kinds of basis functions available in mathematics, whose combinations can well approximate any unknown function. The distortion, on which even little might be known, can thus be modeled by using a linear combination of specific basis functions. The unknown coefficients of the linear combination are computed in the adjustment process. Or in a loose sense, the coefficients are fixed by the noisy image measurements during the least-squares adjustment. It quite resembles the problem of least-squares fitting to the irregular spaced data in mathematics (for the mathematical materials on curve fitting, the readers are referred to textbooks such as Rao (2001)). Therefore, photogrammetric self-calibration can – to a very large extent – be considered as a *function approximation* or, more precisely, a *curve fitting* problem in mathematics. Note that the self-calibration model based on mathematical approximation principle is independent of the physical sources of distortion.

Then, it needs to find a proper group of basis functions for the self-calibration purpose. Various mathematical basis functions can be taken into account. We start with algebraic polynomials because of the historical prevalence of the polynomial APs.

2.1. Polynomial APs

Based on the standard 60% forward overlapping level and the 3×3 and 5×5 grid pattern of image point distribution, Ebner (1976) and Grün (1978) proposed the polynomial APs of second and fourth orders, respectively. Although they constructed their polynomial APs without using explicitly the function approximation principle, we showed that both sets of APs can be exactly derived from this principle (Tang et al., in press). The mathematical theory behind them is the well-defined Weierstrass Theorem, which indicates that any function can be approximated with arbitrary accuracy by a polynomial of sufficiently high degree (Oliver et al., 2010). Both sets of APs can compensate image distortion because polynomials can approximate the unknown distortion function. From the approximation principle, it can be easily explained, that the regular grid pattern is not a prerequisite for applying both APs. The main effect of the irregular image point distribution is degrading correlations but not decaying calibration. This is the exact reason why the Ebner and Grün models still work quite well in the cases where the ideal regular grid pattern is unsatisfied.

In fact, we have developed in our previous work a new family of so-called *Legendre self-calibration APs*, which is established on the basis of the orthogonal univariate Legendre Polynomials. The mathematical relations were derived between Legendre APs and those two historical polynomial APs. From both theoretical analyses and practical test results, Legendre APs can be considered as the superior generalization of the Ebner and Grün models in many senses. The Legendre APs of second and fourth orders should be preferred to the Ebner and Grün models, respectively (Tang et al., in press).

However, there is an intrinsic deficiency of all polynomial APs, including the Ebner, Grün and Legendre APs. All polynomial APs need to eliminate four highly correlated parameters (another two of six, i.e. the two constant terms, are eliminated because they are merely principal point shifts. The removal of them is NOT caused by high correlations). The elimination imposes four constraints on polynomial APs. For example, the constraints on Legendre APs (see Appendix A) are shown in,

$$(\Delta x): +a_1p_{1,0} + a_2p_{0,1} + a_3p_{2,0} + a_4p_{1,1}$$
$$(\Delta y): +a_2p_{1,0} - a_1p_{0,1} - a_3p_{1,1} - a_4p_{0,2} \quad (2)$$

This occurs exactly to the Ebner and Grün models as well. These four constraints, which are caused by high correlations, violate the mathematical principle of polynomial APs. According to the approximation theory and the Weierstrass Theorem, all APs in Δx should be fully independent of those in Δy. In other words, the theoretical number of the unknowns in the Ebner, Grün and fifth order Legendre APs should be 16, 48 and 70, rather than 12, 44 and 66, respectively. The elimination, which is indispensable for constructing polynomial APs, degrades the rigorousness of polynomial APs. It may produce negative effects in calibration.

Therefore, we suggest that algebraic polynomials could be inappropriate for constructing self-calibration models. Our view was shared in Ziemann (1986) that algebraic polynomials "are undesirable from a mathematical point of view because of the high correlation between the different terms". Another inconvenience of polynomial APs is that they usually need more unknown parameters for calibration, compared with other counterparts (the Brown (1976) model of 21 parameters, for example).