ISPRS Journal of Photogrammetry and Remote Sensing 66 (2011) 809–817

Contents lists available at SciVerse ScienceDirect

ISPRS Journal of Photogrammetry and Remote Sensing

journal homepage: www.elsevier.com/locate/isprsjprs

Direct relative orientation with four independent constraints

Yongjun Zhang *, Xu Huang, Xiangyun Hu, Fangqi Wan, Liwen Lin

School of Remote Sensing and Information Engineering, Wuhan University, No. 129 Luoyu Road, Wuhan 430079, PR China

ARTICLE INFO

Article history:
Received 30 April 2010
Received in revised form 7 September 2011
Accepted 7 September 2011
Available online 5 October 2011

Keywords:
Direct relative orientation
Essential matrix
Constraint
Accuracy analysis
Least squares adjustment

ABSTRACT

Relative orientation based on the coplanarity condition is one of the most important procedures in photogrammetry and computer vision. The conventional relative orientation model has five independent parameters if interior orientation parameters are known. The model of direct relative orientation contains nine unknowns to establish the linear transformation geometry, so there must be four independent constraints among the nine unknowns. To eliminate the influence of over parameterization of the conventional direct relative orientation model, a new relative orientation model with four independent constraints is proposed in this paper. The constraints are derived from the inherent orthogonal property of the rotation matrix of the right image of a stereo pair. These constraints are completely new as compared with the known literature. The proposed approach can find the optimal solution under least squares criteria. Experimental results show that the proposed approach is superior to the conventional model of direct relative orientation, especially at low altitude and close range photogrammetric applications.

1. Introduction

Relative orientation is the process of recovering the position and orientation of one image with respect to the other in a certain image space coordinate system (Läbe and Förstner, 2006). It is a classic topic in both photogrammetry and computer vision communities (Huang and Faugeras, 1989; Faugeras and Maybank, 1990; Wang, 1990; Philip, 1996; Zhang, 1998; Mikhail et al., 2001; Stewénius et al., 2006). A pioneer eight point algorithm of relative orientation in computer vision, which is quite similar to the conventional model of direct relative orientation in photogrammetry, is proposed by Higgins (1981) although constraints among the eight unknowns are not considered. Many attentions are concentrated on resolving the fundamental or essential matrix with five conjugate points (Huang and Faugeras, 1989; Faugeras and Maybank, 1990; Philip, 1996; Nistér, 2004; Stewénius et al., 2006). Faugeras and Maybank (1990) proved that the five point algorithm has at most 10 solutions. Because of noises in the image coordinates, the essential matrix will not be exactly decomposable. This may introduce large errors in the estimation of rotation and translation parameters. Better accuracy can be achieved if the decomposability constraints are imposed (Huang and Faugeras, 1989). An iterative method was used by Horn (1990) to determine baseline and rotation parameters with an initial guess of rotation angles. However, initial guesses of rotation angles are not always reasonable in all cases especially at low altitude and close range applications. Nistér (2004) and Stewénius et al. (2006) improved the five point algorithm so that it can operate correctly even in the case of critical surfaces. The five point algorithm proposed by Stewénius et al. (2006) includes six steps. Processes of establishing linear equations from the epipolar constraint, building up 10 third-order polynomial equations with the rank and trace constraint, computing the Gröbner basis on the 10×20 matrix, computing the 10×10 action matrix, parameter back-substitution with the left eigenvectors of the action matrix and five parameter decomposition are used to compute the five relative orientation parameters. Their experiments with large point sets show that the five point algorithm provides the most consistent results.

Relative orientation is also the prerequisite of bundle adjustment (Kraus, 1993; Mikhail et al., 2001; Nistér, 2004), which can achieve the best accuracy of the data (Triggs et al., 2000; Alamouri et al., 2008). However, bundle adjustment often could not obtain the globally optimal solution with inaccurate approximations of unknowns, especially when there are some outliers (Stewénius et al., 2006). There are also methods on the absolute pose determination and 3D reconstruction with point and line correspondences (Liu et al., 1990; Kumar and Hanson, 1994; Taylor and Kriegman, 1995; Li et al., 2007). In the case of photographic configurations with large oblique angles, such as low altitude and close range applications, the approximate angular elements cannot be set as zero. Therefore, direct relative orientation which needs no initial values becomes one of the best choices. However, due to the correlation characteristics among unknowns, it usually brings down the accuracy of relative orientation and even leads to erroneous solutions in some bad geometric configurations (Stewénius et al., 2006).

* Corresponding author.
E-mail address: zhangyj@whu.edu.cn (Y. Zhang).

0924-2716/$ - see front matter © 2011 International Society for Photogrammetry and Remote Sensing, Inc. (ISPRS) Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.isprsjprs.2011.09.006

Usually, homogeneous algebraic representation and singular value decomposition strategy are used in most relative orientation algorithms of computer vision, while error equation of mathematic model and iterative least squares adjustment are used in photogrammetry. In this paper, a new direct relative orientation model from the photogrammetric point of view is proposed. Different from the epipolar constraint of essential matrix in computer vision communities that taking normalized image points as observations, original focal plane coordinates of conjugate points and the corresponding focal lengths are used in the linear model. The new model is composed of four constraints together with the conventional model of direct relative orientation. The four constraints are derived from the inherent orthogonal property of rotation matrix. Overview of conventional direct relative orientation is given in the next section. Principle of ill-posed problem that caused by over parameterization is discussed in Sections 3. The proposed new model of direct relative orientation with constraints and practical solving procedures are thoroughly addressed in Sections 4 and 5, respectively. Then several experiments are performed and compared with the ground truth in detail. Finally, conclusions are briefly outlined.

2. Conventional model of direct relative orientation

Given two images of a scene taken from different viewpoints, a stereo model can be created to reestablish the original epipolar geometry. The mathematic model of relative orientation can be expressed by coplanarity equation (Wang, 1990; Mikhail et al., 2001):

$$\mathbf{F} = \begin{vmatrix} B_x & B_y & B_z \\ u & v & w \\ u' & v' & w' \end{vmatrix} = 0 \tag{1}$$

where B_x, B_y and B_z are baseline parameters of a stereo pair, $(u\ v\ w)^T = (x\ y\ -f)^T$ and $(u'\ v'\ w')^T = \mathbf{R} \cdot (x'\ y'\ -f')^T$ coordinates of conjugate points in the image space coordinate system, (x, y), (x', y') the original focal plane coordinates of conjugate points, $\mathbf{R} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$ the rotation matrix composed of three angles φ, ω, κ, f and f' the focal lengths of two images, respectively.

Eq. (1) can be transformed into the following linear form of Eq. (2). It is similar to the basic equation of fundamental or essential matrix (Hartley and Zisserman, 2000) used in computer vision, except that f and f' explicitly exist in the following equation.

$$L_1yx' + L_2yy' - L_3yf' + L_4fx' + L_5fy' - L_6ff' + L_7xx' + L_8xy' - L_9xf' = 0 \tag{2}$$

where

$$\begin{aligned} &L_1 = B_x \cdot c_1 - B_z \cdot a_1 \quad && L_2 = B_x \cdot c_2 - B_z \cdot a_2 \quad && L_3 = B_x \cdot c_3 - B_z \cdot a_3 \\ &L_4 = B_x \cdot b_1 - B_y \cdot a_1 && L_5 = B_x \cdot b_2 - B_y \cdot a_2 && L_6 = B_x \cdot b_3 - B_y \cdot a_3 \\ &L_7 = B_z \cdot b_1 - B_y \cdot c_1 && L_8 = B_z \cdot b_2 - B_y \cdot c_2 && L_9 = B_z \cdot b_3 - B_y \cdot c_3 \end{aligned} \tag{3}$$

The coefficients $L_i (i = 1, 2, \ldots, 9)$ in Eq. (2) can only be determined up to a scale, so there are eight independent parameters. Different from known methods in computer vision communities that usually set the last element of fundamental matrix as 1.0, usually L_5 of Eq. (2) is set to be 1.0 in photogrammetry for the convenience of setting up error equations and minimizing vertical parallaxes. Suppose $L_i^0 = L_i/L_5 (i = 1, 2, \ldots, 9)$ and $L_5^0 = 1$, then Eq. (2) becomes:

$$L_1^0yx' + L_2^0yy' - L_3^0yf' + L_4^0fx' + fy' - L_6^0ff' + L_7^0xx' + L_8^0xy' - L_9^0xf' = 0 \tag{4}$$

It is the basic mathematic model of conventional direct relative orientation. This model can be used to directly calculate the eight unknowns $L_1^0, L_2^0, L_3^0, L_4^0, L_6^0, L_7^0, L_8^0, L_9^0$ without initial values. The baseline parameter B_x has no influence on building up a stereo model, it can be assumed to be known, for example the mean x-parallax. So parameters, B_y and B_z can be obtained by the following equations:

$$\begin{aligned} &L_5^2 = 2B_x^2/(L_1^{0^2} + L_2^{0^2} + L_3^{0^2} + L_4^{0^2} + L_5^{0^2} + L_6^{0^2} - L_7^{0^2} - L_8^{0^2} - L_9^{0^2}) \\ &L_i = L_i^0 \cdot L_5 (i = 1, 2, \cdots, 9) \\ &B_y = -(L_1L_7 + L_2L_8 + L_3L_9)/B_x \\ &B_z = (L_4L_7 + L_5L_8 + L_6L_9)/B_x \end{aligned} \tag{5}$$

Elements of the rotation matrix $\mathbf{R}$ can be computed by Eqs. (3) and (5). Finally, three rotation angles can be decomposed by the definition of $\mathbf{R}$ (Wang, 1990; Mikhail et al., 2001). There are two sets of possible solutions about the rotation angles ϕ, ω, κ. One of the two solutions is the true configuration, another one is the twisted pair by rotating the right image 180 degrees around the baseline. It is very easy to find the correct solution with the fact that the photographic object is in front of the camera.

3. Ill-Posed problem caused by over parameterization

Over parameterization usually results in ill-posed problem when constraint relationships among unknowns are not considered (Faugeras and Maybank, 1990). Small errors in observations may be enlarged and therefore the solution is often seriously biased from the ground truth. Suppose there is a mathematic model with the following form:

$$F_1(\mathbf{X_F}) = 0 \tag{6}$$

where $\mathbf{X_F}$ is a n-dimensional vector, i.e., there are n independent parameters in the above model. A new m-dimensional $m \geqslant n$ vector $\mathbf{Y_F}$ can be obtained by applying a certain transformation $\mathbf{Y_F} = T(\mathbf{X_F})$. So we can get the following model that takes $\mathbf{Y_F}$ as parameter:

$$F_2(\mathbf{Y_F}) = 0 \tag{7}$$

The model $F_2(\mathbf{Y}_F = 0)$ is over parameterized in the case of $m > n$. There must be $m - n$ conditional equations $G(\mathbf{Y_F}) = 0$ among the elements of vector $\mathbf{Y_F}$. It is a typical model of adjustment with functional constraints when solving the equations $F_2(\mathbf{Y_F}) = 0$ and the conditional equations $G(\mathbf{Y_F}) = 0$. The unbiased least squares solution is:

$$\mathbf{Y_1} = \mathbf{N}_{BB}^{+}\mathbf{B_F^T l_F} - \mathbf{N}_{BB}^{+}\mathbf{C_F^T N}_{CC}^{-1}\mathbf{C_F N}_{BB}^{+}\mathbf{B_F^T l_F} - \mathbf{N}_{BB}^{+}\mathbf{C_F^T N}_{CC}^{-1}\mathbf{W_F} \tag{8}$$

where $\mathbf{Y_1}$ is the solution of the unknown vector $\mathbf{Y_F}$, $\mathbf{N}_{BB}^{+}$ is the pseudo-inverse (Hartley and Zisserman, 2000) of $(\mathbf{B_F^T B_F})$, i.e., $\mathbf{N}_{BB}^{+} = (\mathbf{B_F^T B_F})^{+}$, $\mathbf{N}_{CC}^{-1} = (\mathbf{C_F N_{BB}^{+} C_F^T})^{-1}$, $\mathbf{B_F}$ is the design matrix of the unknown vector $\mathbf{Y_F}$, $\mathbf{C_F}$ is the coefficient matrix of the unknown vector $\mathbf{Y_F}$ corresponded to the conditional equations $G(\mathbf{Y_F}) = 0$, $\mathbf{l_F}$ and $\mathbf{W_F}$ are the vectors of constant items of the two kinds of equations which can be calculated by the observations and the approximate values of unknowns. The approximate value of $\mathbf{Y_F}$ has to be provided. Usually it can be set to be zero in linear cases.

However, the adjustment model to solve $F_2(\mathbf{Y_F}) = 0$ is a typical model of adjustment of observation equations if the condition $G(\mathbf{Y_F}) = 0$ is not considered. The least squares solution is just the first item of the right side of Eq. (8), i.e., $\mathbf{Y_2} = \mathbf{N_{BB}^{+} B_F^T l_F}$. In the case of over parameterization, especially when there are also some outliers in observations, the least squares solution can be a biased estimation if the constraints among the unknown parameters are not been considered. The conventional model of direct relative orientation is obviously over parameterized, which means the achieved solution is not always reliable.